An Investigation of Peak-to-Average Power Reduction in MIMO-OFDM Systems

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ABSTRACT

Multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) technology is one of the most attractive candidates for fourth generation (4G) mobile radio communication. It effectively combats the multipath fading channel and improves the bandwidth efficiency. At the same time, it also increases system capacity so as to provide a reliable transmission. However, the main drawback of MIMO-OFDM system is high peak-to-average power ratio (PAPR) for large number of sub-carriers, which result in many restrictions for practical applications. Coding, phase rotation and clipping are among many PAPR reduction schemes that have been proposed to overcome this problem. In this thesis, we will mainly investigate the PAPR reduction performance with two different PAPR reduction methods: partial transmit sequence (PTS) and selective mapping (SLM). These two methods are sub-entities of phase rotation scheme. In addition, several corresponding modified algorithms are studied with respect to balanced performance and applicability.

Keywords: multiple input multiple output (MIMO), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), selected mapping (SLM), partial transmit sequence (PTS).
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# Table of Contents

ABSTRACT .................................................................................................................. I

ACKNOWLEDGEMENTS ............................................................................................. III

Chapter 1 ...................................................................................................................... 1

Introduction ................................................................................................................... 1

1.1 Background of the Problem .................................................................................... 1

1.2 Problem Statement ................................................................................................. 2

1.3 Thesis Structure ...................................................................................................... 2

Chapter 2 ...................................................................................................................... 4

MIMO-OFDM SYSTEMS ............................................................................................. 4

2.1 MIMO System ........................................................................................................... 4

2.1.1 Basic Structure of MIMO System ...................................................................... 4

2.1.2 MIMO Channel and Capacity ........................................................................... 5

2.1.3 MIMO Space-time Code .................................................................................... 8

2.2 Orthogonal Frequency Division Multiplexing (OFDM) ......................................... 9

2.2.1 Single Carrier and Multicarrier System ............................................................. 9

2.2.2 FDMA and OFDM System ................................................................................ 11

2.2.3 Basic Structure of OFDM System ..................................................................... 12

2.2.4 Orthogonality of OFDM System ....................................................................... 14

2.2.5 Cyclic Prefix of OFDM System ....................................................................... 16

2.2.6 Advantage and Disadvantage of OFDM ............................................................. 17

2.3 MIMO-OFDM System .......................................................................................... 18

2.3.1 Basic Structure of MIMO-OFDM System ......................................................... 19

2.4 Peak-to-Average Power Ratio in OFDM System .................................................... 20

2.4.1 PAPR Definition ............................................................................................... 20

2.4.2 Probability Distribution Function of PAPR ...................................................... 23

2.4.3 Influencing Factors of PAPR ............................................................................ 24

2.2.4 Studying of PAPR Reduction Techniques ....................................................... 27

Chapter 3 .................................................................................................................... 29

Research on Signal Scrambling Techniques ................................................................. 29

3.1 Selected Mapping Method ..................................................................................... 30

3.1.1 Principle of SLM (Selected Mapping) .............................................................. 31

3.1.2 Simulation of SLM Scheme .............................................................................. 34

3.2 Partial Transmit Sequence ...................................................................................... 37

3.2.1 Principle of PTS (Partial Transmit Sequence) .................................................. 37

3.2.2 Modified PTS Scheme ..................................................................................... 39

3.2.3 Simulation of PTS Scheme .............................................................................. 44

IV
Chapter 1

Introduction

1.1 Background of the Problem

Nowadays, third generation (3G) mobile communication systems have became popular all around the world. However, its services cannot provide a very big dynamic range of data rates, nor can it meet the requirements of a variety of business types. Besides, voice transportation in 3G still relies on circuit switching technology, which is the same method as used in second-generation (2G) communication systems, rather than pure Internet Protocol (IP) approach. Thus, based on consideration listed above, many countries have already carried out research on the next completely evolutionary fourth generation (4G) communication systems which provide a comprehensive and secure IP solution where voice, data, and multimedia can be offered to users at "anytime, anywhere" with higher data rates than previous generations [1].

Since bandwidth resource in 4G mobile communications is still scarce, in order to improve spectrum efficiency and achieve as high as 100Mbps wireless transmission rate, it requires more advanced techniques to be employed. The limitation of modulation schemes in existing communication systems has become an obstruction in further increasing the data rate. Hence, next generation mobile communication systems need more sophisticated modulation scheme and information transmission structure.

Multiple input multiple output (MIMO) and orthogonal frequency division multiplexing (OFDM) have therefore been adopted due to their superior performance. They promise to become key high-speed wireless communication technologies and combining them can provide wireless industry evolution from 3G to 4G system.
1.2 Problem Statement

In MIMO-OFDM system, the output is the superposition of multiple sub-carriers. In this case, some instantaneous power outputs might increase greatly and become far higher than the mean power of the system when the phases of these carriers are same. This is also defined as large Peak-to-Average Power Ratio (PAPR).

High PAPR is one of the most serious problems in MIMO-OFDM system. To transmit signals with high PAPR, it requires power amplifiers with very high power scope. These kinds of amplifiers are very expensive and have low efficiency-cost. If the peak power is too high, it could be out of the scope of the linear power amplifier. This gives rise to non-linear distortion which changes the superposition of the signal spectrum resulting in performance degradation. If there are no measures to reduce the high PAPR, MIMO-OFDM system could face serious restriction for practical applications.

To combat high PAPR, one intuitive solution is to adopt amplifiers to have larger trade-off range. However, these types of amplifiers are generally expensive and have low efficiency-cost, and therefore are of no practical use. On the other side, certain algorithms were introduced and been proved have a good performance of high PAPR reduction. Hence, in this thesis, some currently promising PAPR reduction methods are studied and compared. The performance of these reduction schemes are evaluated by using simulation software, Matlab.

1.3 Thesis Structure

This thesis is organized as follows:
1. Chapter 2 starts out with the introduction of MIMO and OFDM technology respectively. That includes their basic structure and technical essential. Then, the PAPR problem and its definition are given. In the end, three distinctive types of PAPR reduction techniques are introduced as a lead-in to Chapter 3 in terms of basic principle and algorithm overhead.

2. In Chapter 3, we focus our research on signal scrambling techniques. Two sub-type algorithms, selected mapping (SLM) and partial transmit sequence (PTS) are investigated.
A comprehensive analysis and comparison are conducted in terms of all possible influencing factors and PAPR reduction performance, respectively. Some research findings are obtained based on the simulation results. At last, we also compare SLM and PTS algorithm with respect to auxiliary information and PAPR reduction performances.

3. In Chapter 4, a conclusion of thesis is drawn and some suggestions are provided for the future work.

4. In Appendix A, three flow diagrams of related Matlab routines are given and in Appendix B, corresponding routines are provided with detailed comments.
Chapter 2

MIMO-OFDM SYSTEMS

2.1 MIMO System

MIMO signaling is a groundbreaking development pioneered by Jack Winters of Bell Laboratories in his 1984 article [2]. Several different antenna configurations are used in defining space-time systems.

2.1.1 Basic Structure of MIMO System

There exist several communication transmission models as follows (see Fig. 2.1):

1. Single-input-and-single-output (SISO) system: It uses only one antenna both at the transmitter and receiver.


3. Multiple-input-and-single-output (MISO) system: It has multiple transmitting antennas and one receiving antenna.

4. Multiple-input-multiple-output (MIMO) system: It uses multiple antennas both for transmission and reception. Multiple transmitting and receiving antennas will achieve antenna diversity without reducing the spectral efficiency.

In MIMO system, a number of antennas are placed at the transmitting and receiving ends, their distances are separated far enough. The distance between different base station antennas can be set as 10 times the carrier wavelength and mobile station antennas can be separated by half carrier wavelength. In this way, independent channels between the transmitting and receiving ends are formed so as to achieve spatial diversity or space division multiplexing.
The idea is to realize spatial multiplexing and data pipes by developing space dimensions which are created by multi-transmitting and receiving antennas. The block diagram in Fig. 2.1 illustrates the antenna configuration is space-time systems.

![Block Diagram](image)

Figure 2.1 Different antenna configurations in space-time systems.

### 2.1.2 MIMO Channel and Capacity

Generally, there are several kinds of channel impairments in wireless communication.

1. Free space path loss (FSPL): This refers to power loss of electromagnetic wave when there is an unobstructed line-of-sight path exists between transmitter and receiver. The free space power received by receiving antenna, which is separated from transmitting antenna by a distance $d$, is given by

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$  \hspace{1cm} (2-1)

Where $P_t$ is transmitted power, $P_r(d)$ is received power, $G_t$ is transmitting antenna gain, $G_r$ is receiving antenna gain, $d$ is the transmitter-receiver (T-R) separation distance in meters, $L$ is the system loss factor (not related to propagation, $L \geq 1$) and $\lambda$ is the wavelength in meters. The gain of an antenna is related to its effective aperture by
\[ G = \frac{4\pi A_e}{\lambda^2} \]  

where \( \lambda \) is wavelength of carrier and \( \lambda = \frac{c}{f} \), \( f \) is carrier frequency in Hz and \( c \) is the speed of light in meters/sec (3 \times 10^8 \text{ m/sec}), \( A_e \) is effective aperture of antenna in m².

2. Shadow fading: In reality, big obstructions such as hills or large buildings obscure the main signal path between the transmitter and the receiver, which will lead to shadowing and amplitude fluctuation of receiving signals. In fact, free-space path loss and shadow fading belong to large-scale fading or slow fading.

3. Small-scale fading (multipath): This refers to rapid fluctuation of amplitude and phase over a short period of time or travel distance. Small-scale fading can be generally related to reflection, diffraction and scattering. The interaction of these three different propagation mechanisms cause fading at a specific location [4].

Rayleigh fading is an applicable model to describe small-scale fading when there is no dominant propagation along the line of sight between the transmitter and receiver. If there is a dominant propagation along the line of sight, Rician fading may be more applicable.

System capacity can be defined as maximum transmission data rate in condition of small error probability. Both Telatar and Foschini in their papers state that in MIMO system, system capacity can be increased linearly by means of setting up multiple space sub-channels which connect the transmitter and receiver [5] [6].

The transmitted signal bandwidth is so narrow that its frequency response can be considered as being flat. Define the channel matrix \( \mathbf{H} \) is \( N_r \times N_t \) complex matrix, elements of it are fading coefficients from the \( j-th \) transmit antenna to the \( i-th \) receive antenna. Assuming that a MIMO system with a transmit array of \( N_T \) antennas and a receive array of \( N_R \) antennas, the transmission can be expressed as:

\[ \mathbf{y} = \mathbf{Hx} + \mathbf{n}, \]  

where \( \mathbf{y} \) is \( N_R \times 1 \) receiving vector, \( \mathbf{x} \) is \( N_T \times 1 \) transmitting vector, \( \mathbf{n} \) is additive white Gaussian noise which autocorrelation matrix \( \mathbf{R}_n = E\{\mathbf{nn}^H\} = N_0 \mathbf{I}_{N_T} \). \( \mathbf{I}_{N_T} \) is an \( N_T \times N_T \) identity matrix.

$N_T$ identity matrix, $N_0$ is identical noise power of each receiving branch. Assuming that the signals transmitted by separated antennas are independent of each other and for each one its mean value equals to 0 and variance equals to 1. The black diagram of flat fading MIMO channel is shown in Fig. 2.2.

![Flat fading MIMO channel](image)

Figure 2.2 Flat fading MIMO channel.

MIMO system capacity can be expressed as [7]:

$$C = \max_{T_r(R_{ss}) = N_T} \left\{ \log_2 \left[ \det \left( I_{N_R} + \frac{E_s}{N_T N_0} H R_{ss} H^H \right) \right] \right\}$$

(2-4)

The unit of system capacity $C$ is bit/s/Hz, $\det (\cdot)$ denotes matrix determinant. If channel knowledge is unknown for the transmitter, and the signals transmitted from each antenna have equal powers, that is, $R_{ss} = I$. In this situation, the system capacity can be rewritten as [7]:

$$C = \log_2 \left[ \det \left( I_{N_R} + \frac{E_s}{N_T N_0} H H^H \right) \right]$$

(2-5)

The following conclusions can be drawn on the basis of these formulas: Multi-antenna system has indeed improved the channel capacity compared to traditional single-antenna system. The increased channel capacity can be used to purely raise the information transmission rate or improve the reliability of communications systems by enhancing information redundancy in
condition of maintaining the information transmission rate. Normally, a tradeoff can be made by combining these two methods to form a mixed scheme.

### 2.1.3 MIMO Space-time Code

The space-time coding technique is essentially a two-dimensional space and time processing method. While multiple antennas both for transmission and reception are used to improve wireless communication systems capacity and data rate in space-domain. In time-domain, different signals can be transmitted at different time slots using the same antenna at the same time. Correlation of time and space is introduced between signals which are transmitted by different antennas so that the receiver antennas can realize diversity reception. Therefore, space-time coding is especially meant for higher coding gain without using more bandwidth which effectively enhances capacity of wireless systems.

MIMO system can be generally divided into Space-Time Coding (STC) and Spatial Multiplexing (SM). Classical STC includes Space-Time Trellis Code (STTC) and Space-Time Block Code (STBC). STTC achieves full diversity over fading channel and offers a good coding gain. STBC can also achieve a full diversity gain by performing a simple maximum likelihood decoding algorithm. A typical SM technique is Layered Space-Time Structure, which was first proposed by Bell Labs (BLAST). It includes V-BLAST, H-BLAST and D-BALST. The most basic form is V-BLAST architecture, which is widely used in the flat fading channel, but it cannot obtain spatial diversity gain.
2.2 Orthogonal Frequency Division Multiplexing (OFDM)

The concept of orthogonal frequency division multiplexing (OFDM) first appeared in the 1950s. It has nearly 60 years of development history.

2.2.1 Single Carrier and Multicarrier System

1. Basic structure of a single carrier system

In a single carrier system, signals are pulse-formed by a transmitter filter $h_t(t)$ before being applied to a multipath channel. At the receiver, the incoming signal is passed through a receiving match filter $h_r(t)$ to maximize the signal-to-noise ratio (SNR). The basic structure diagram of a single carrier system is shown in Fig. 2.3.

![Figure 2.3 Basic structure of a single carrier system.](image)

2. Basic structure of a multicarrier system

In a multicarrier system, input signals which are divided by a multiplexer are applied to pulse-formed $h_t(t)$ filters before being transmitted through multipath environment. Correspondingly, the receiving ends consist of $N$ parallel paths. Each one is passed through a respective match filter $h_r(t)$ to realize maximum SNR. The basic structure diagram of a single multicarrier system is shown in Fig. 2.4.
In a classical wireless communication model, the transmitted signal arrives at the receiver via various paths. Thus, extracting the original signal at the receiving end becomes extremely difficult. If the signal is transmitted at time intervals $T$, then the parameter concerning the multipath channel is the delay $\tau_{\text{max}}$ of the longest path with respect to the earliest path. The received signal can be theoretically influenced by $\frac{\tau_{\text{max}}}{T}$ previous signals, which must be considered seriously by receiver [8].

![Figure 2.4: Basic structure of a multicarrier system.](image)

In a single carrier system, it is assumed that transmission rate $R = \frac{1}{T}$ and maximum channel delay is $\tau_{\text{max}}$. In a multicarrier system, the original data stream of rate $R$ is multiplexed into $N$ parallel data streams with rate $R_{\text{mc}} = \frac{1}{T_{\text{mc}}} = \frac{R}{N}$. Each of the substreams is modulated with a different subcarrier frequency and all the data streams are transmitted in the same band. In this case, the ISI of each sub-system reduces to $\frac{\tau_{\text{max}}}{T_{\text{mc}}} = \frac{\tau_{\text{max}}}{N \cdot T}$. As the value of $N$ increases, inter-symbol interference (ISI) becoming decreases.

In a single carrier system, fading or interference can make the entire link fail. However, in a multicarrier system, only a small part of subcarriers will be affected. Error correction coding methods can be employed to correct the errors which were happened in subcarriers. OFDM is a special form of multicarrier modulation (MCM), in which a signal is transmitted over a number of lower rate subcarriers.
2.2.2 FDMA and OFDM System

In the traditional frequency division multiplexing (FDM) system, signals are transmitted in different channels. Guard intervals are needed for channel isolation and filtering so as to prevent interference and guarantee an effective wireless communication. However, at the receiving end, a series of band-pass filters are needed to separate and extract information, which results in low frequency spectrum utilization. Up to 50% of the total spectrum is wasted in this manner. Another problem is that realization of filter banks is not easy. Fig. 2.5 shows the frequency spectrum utilization efficiency in FDM and OFDM system, respectively.

Figure 2.5 Comparison of frequency spectrum in FDM and OFDM system.

![Spectrum of traditional FDM modulation scheme](image1)

![Spectrum of OFDM multicarrier modulation scheme](image2)

OFDM technology overcomes this problem by dividing available bandwidth into a number of sub-channels. Sub-channels are made orthogonal to one another. OFDM signal consists of $N$ sub-carriers, which are transmitted with equal interval. Each subcarrier has a null at the center frequency of the adjacent carrier, which results in zero interference among the carriers. The frequency spectrum is $1/2$ overlapped. Fig. 2.6 displays OFDM signal in frequency domain.

![Figure 2.6 Frequency spectrum of OFDM signal.](image3)
Each subcarrier in OFDM system signal has a very narrow bandwidth with low symbol rate. The signal therefore, has immunity on multipath delay spread. At the receiving end, correlation modulation technique can be employed to separate different sub-carriers, thereby avoiding the use of filter banks and increasing spectrum utilization. Since OFDM signal is transmitted in low-speed parallel subcarriers, it has increased symbol period which help to reduce the time dispersion and ISI of the system.

2.2.3 Basic Structure of OFDM System

A typical OFDM transmission system is shown in Fig. 2.7. At the transmitting end, first of all, input binary serial data stream is first processed by channel encoder, constellation mapping and serial to parallel (S/P) conversion. A single signal is divided into $N$ parallel routes after $N$-point inverse fast Fourier transform (IFFT). Each orthogonal sub-carrier is modulated by one of the $N$ data routes independently. By definition the $N$ processed points constitute one OFDM symbol.

Next, convert modulated parallel data to serial sequence and then copy the last $L$ samples of one symbol to the front as cyclic prefix (CP). At last, arrive at transmitter after process of digital to analog (D/A) conversion and radio frequency (RF) modulation. To recover the information in OFDM system, reception process is converse and self-explanatory. At the receiving end, digital down conversion is carried out, demodulate receiving signals.

At last, demodulated signals are fed into an analog to digital (A/D) converter, sample output and take timing estimation to find initial position of OFDM symbol. The CP added in transmission process is removed and $N$-Points fast Fourier transform (FFT) transformation will be conducted on the left sample points to recover the data in frequency domain. The output of baseband demodulation is passed to a channel decoder, which eventually recover the original data.

An OFDM symbol is made of sub-carriers modulated by constellations mapping. This mapping can be achieved from phase-shift keying (PSK) or quadrature amplitude modulation (QAM). For an OFDM system with $N$ sub-carriers, the high-speed binary serial input stream is denoted as $\{a_i\}$. After serial to parallel (S/P) conversion and constellation mapping, a new parallel signal sequence $\{d_0, d_1, \cdots, d_i, \cdots, d_{N-1}\}$ is obtained, $d_i$ is a discrete complex-valued
signal.

Here, \( d_i \in \{\pm 1\} \) when BPSK mapping is adopted. When QPSK mapping is used, \( d_i \in \{\pm 1, \pm j\} \). Each element of parallel signal sequence is supplied to \( N \) orthogonal sub-carriers \( \{e^{j2\pi f_0 t}, e^{j2\pi f_1 t}, \ldots, e^{j2\pi f_{N-1} t}\} \) for modulation, respectively. Finally, modulated signals are added together to form an OFDM symbol. Use of discrete Fourier transform simplifies the OFDM system structure.

![Figure 2.7 Basic structure of OFDM system.](image)

An OFDM receiver consists of a group of decoders, which move different carrier frequencies to zero frequency and perform integration over one symbol period. Since sub-carriers are orthogonal to one another, only specified carrier can be demodulated, the rest irrelevant carriers do not have any impact on the results of the integration.

The frequency of an OFDM signal can be expressed as

\[
f_i = f_c + i \cdot \Delta f
\]

(2-6)

where \( f_c \) stands for carrier frequency, \( \Delta f \) is the smallest interval between different sub-carrier frequencies. \( \Delta f \) is given by

\[
\Delta f = \frac{1}{T} = \frac{1}{N t_s}
\]

(2-7)
where $t_s$ is time interval of symbol sequence $\{d_0, d_1, \ldots, d_i, \ldots, d_{N-1}\}$.

Generally, we use complex baseband equivalent signal to describe OFDM output signal, which can be expressed as follows:

$$s(t) = \sum_{i=0}^{N-1} d_i e^{j2\pi f t} = \sum_{i=0}^{N-1} d_i e^{j2\pi f t}, \quad t \in [0, T]$$

The real and imaginary parts of complex factor corresponding to in-phase components and quadrature components of OFDM symbols, respectively.

At the receiver, corresponding sub-carriers are applied to the input for demodulation. The process of demodulation for the $k$-th sub-carrier signal is described as follows: The output signal is multiplied by the $k$-th demodulation carrier expression $\exp(-j\pi (2k - N)t/T)$, and then integrate the product over one OFDM symbol period $T$. The integration result is the transmitting signal corresponding to the $k$-th sub-carrier signal.

It is known that modulation and demodulation in OFDM system can be achieved by IFFT and FFT, respectively. A data symbol in the “frequency domain” is transformed to “time-domain” by performing the $N$ point IFFT operation, before being sent across to the wireless channel for transmission after radio frequency modulation.

### 2.2.4 Orthogonality of OFDM System

A number of non-zero subcarriers are included in an OFDM symbol period which last $T$ seconds. Therefore, the frequency spectrum of the OFDM symbol can be seen to be a result of convolution between the spectrum of rectangular pulse and a group of sub-carriers at different frequencies. The duration of rectangular pulse is $T$. The spectrum of rectangular pulse is sinc($f \cdot T$). The zero points of this function only take place at integer multiples of $1/T$. For an assigned sub-carrier frequency point, only the corresponding sub-carrier can have a maximum value with all the other sub-carriers taking the value of zero at this point.

Therefore, based on this special property, symbols of each sub-carrier can be extracted from a number of overlapped sub-carriers during the modulation process and without causing any interference effects. Eq. (2-9) shows the mathematical expression for this phenomenon.
The expression for demodulating the $k$-th sub-carrier in Eq. (2-8) can be written as Eq. (2-10). The estimated transmitted discrete complex-valued signal $\hat{d}_k$ is as a result of integration over time $T$:

$$\hat{d}_k = \frac{1}{T} \int_0^T e^{-j2\pi f_m t} \cdot e^{-j2\pi f_n t} \, dt = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

(2-9)

Observing the mathematical derivation above, we learned that for correct demodulation of the $k$-th sub-carrier, the value of integration must equals to 1 with the condition of $i = k$ being satisfied. If $i \neq k$, the power factor of complex integral variable $\frac{i-k}{T}$ is integer multiple of $1 / T$, integration result is equals to 0.

![Four-carrier OFDM signal waveform.](image)

Fig. 2.8 is an example of a four-carrier OFDM signal. In this example, it is assumed that all
the waveforms have the same amplitude and phase. Fig. 2.8 (a) illustrates four independent sub-carrier waveforms in time-domain. Fig. 2.8 (b) shows the waveform of a composite OFDM signal in time domain. Fig. 2.8 (c) illustrates four independent sub-carrier waveforms in frequency domain. Fig. 2.8 (d) shows the waveform of a composite OFDM signal in frequency domain.

As can be seen from the Fig. 2.8, the adjacent sub-carrier has an integer number of cycles over one OFDM symbol period, and the discrepancy among all adjacent subcarriers is one carrier period, which ensures that subcarriers are orthogonal. In the frequency domain, all the overlapped subcarriers undergo a rectangular waveform shaping to generate the frequency spectrum in a form of sinc function.

### 2.2.5 Cyclic Prefix of OFDM System

In OFDM system, the use of Cyclic Prefix (CP) can guarantee orthogonality of signals even when they travel through multi-path channels [8]. To avoid ISI, the condition; \( T_G > T_{\text{max}} \) should be satisfied, where \( T_G \) is the length of CP and \( T_{\text{max}} \) is the maximum delay spread [9].

As shown in Fig. 2.9, a CP is a copy of the last part of a OFDM symbol moved to the front of symbol. Assuming that the number of the extended OFDM symbol is \( N_G \), then the period of a practical OFDM symbol is \( T+T_G \), where \( T \) is cycle for the FFT transform, \( T_G \) is the length of guard interval, which is inserted to suppress ISI caused by multipath distortion. An OFDM symbol including CP can be expresses as follows:

\[
s'_n = s'(t)|_{t=nt_x} = \sum_{i=0}^{N-1} d_i \cdot e^{j2\pi \frac{in}{N}}, n = N_G, \cdots, -1,0,\cdots, N-1
\]  

(2-11)

Operation between the signal and channel changes from linear convolution to cyclic convolution when CP is used with OFDM. In the frequency domain, linear weighing will be used. These changes avoid inter-symbol interference, while ensuring orthogonality among the sub-carriers all the time.
2.2.6 Advantage and Disadvantage of OFDM

OFDM is a widely used communication technique working on combating multipath distortion. OFDM applications have been extended from high frequency (HF) radio communication to telephone networks, digital audio broadcasting and digital television terrestrial broadcasting.

The OFDM technique (multicarrier modulation) compared to traditional single carrier modulation scheme has several advantages:

1. OFDM has very high frequency spectrum efficiency. Since in the OFDM system, sub-carriers are orthogonal to each other, channel spectrum overlapping is allowed, which can utilize limited spectrum resources maximally [10].

2. OFDM system is relatively simple to realize. Modulation and demodulation can be achieved by FFT and IFFT.

3. OFDM has the ability to combat multi-path interference. In OFDM, high-speed serial data streams are transferred to parallel transmission which increases the duration of data symbols carried by corresponding sub-carriers. This effectively reduces the channel time dispersion caused by ISI.

4. OFDM can use a different number of sub-channels to provide different transmission rates between uplink and downlink. Currently, wireless data services are often non-symmetrical,
that is, downlink channels carry more traffic than uplink channels. This requires a physical layer that supports non-symmetric high-speed data transmission.

However, OFDM also has some shortcomings. These are:

1. It is vulnerable to the effects of frequency offset. OFDM sub-carrier spectrums overlap each other, which require strict orthogonality among them. Frequency offset impairs orthogonality and can also degrade demodulation performance. Time offset can lead to OFDM symbol interference and amplitude attenuation.

2. The use of OFDM introduces a high peak-average power ratio. It is known that in OFDM, the output is a superposition of sub-carriers. When the phases of carriers are the same, this could lead to some instantaneous power outputs which are higher than the mean power of the system. This results in a larger Peak-to-Average Power Ratio (PAPR). When the peak power is too high, it could be out of range of linear power amplifiers. This gives rise to non-linear distortion and changes the superposition of the signal spectrum. It also destroys sub-carrier orthogonality and degrades the performance of OFDM.

2.3 MIMO-OFDM System

In high-speed wireless communication, combining MIMO and OFDM technology, OFDM can be applied to transform frequency-selective MIMO channel into parallel flat MIMO channel, reducing the complexity of the receiver, through multipath fading environment can also achieve high data rate robust transmission. Therefore, MIMO-OFDM systems obtain diversity gain and coding gain by space-time coding, at the same time, the OFDM system can be realized with simple structure. Therefore, MIMO-OFDM system has become a welcome proposal for 4G mobile communication systems.
2.3.1 Basic Structure of MIMO-OFDM System

At the transmitting end, a number of transmission antennas are used. An input data bit stream is supplied into space-time coding, then modulated by OFDM and finally fed to antennas for sending out (radiation). At the receiving end, in-coming signals are fed into a signal detector and processed before recovery of the original signal is made. Fig. 2.10 shows the basic structure of a MIMO-OFDM system.

![Figure 2.10 Basic structure of MIMO-OFDM system.](image)

Presently, many companies and research institutions have developed MIMO-OFDM experimental systems. Airbust –production of Iospan Company that first used MIMO and OFDM technology in the physical layer at the same time for wireless communication systems [11].

In MIMO-OFDM system, the frequency response of $k$-th sub-carrier can be expressed as follows:

$$H_k^{(q,p)}(n) = \sum_{l=1}^{L-1} h_l^{(q,p)}(n) W_K^{kl}$$  \hspace{1cm} (2-12)

where $k = 0, ..., K-1$, $h_l^{(q,p)}(n)$ is the impulse response, that is from $p$-th transmitter antenna to $l$-th channel of $q$-th receiver antenna. $n$ is sequence number of the symbol and $K$ is the total number of sub-carriers.

Assuming that $W_K = e^{-j2\pi/K}$, while $M$ and $N$ are the total number of transmitter and receiver
antennas. The output response for the $q$-th receiver antenna can be written as:

$$y_k^{(q)}(n) = \sum_{p=1}^{M} H_k^{(q,p)}(n)x_k^{(p)}(n) + \zeta_k(n)$$  \hspace{1cm} (2.13)$$

$q=1,\ldots, N; \ k=0,\ldots, K-1$ and $\zeta_k(n)$ is Gaussian noise with variance $\delta_n^2$.

### 2.4 Peak-to-Average Power Ratio in OFDM System

The instantaneous output of an OFDM system often has large fluctuations compared to traditional single-carrier systems. This requires that system devices, such as power amplifiers, A/D converters and D/A converters, must have large linear dynamic ranges. If this is not satisfied, a series of undesirable interference is encountered when the peak signal goes into the non-linear region of devices at the transmitter, such as high out of band radiation and inter-modulation distortion. PAPR reduction techniques are therefore of great importance for OFDM systems [12].

#### 2.4.1 PAPR Definition

Theoretically, large peaks in OFDM system can be expressed as Peak-to-Average Power Ratio, or referred to as PAPR, in some literatures, also written as PAR. It is usually defined as [13]:

$$PAPR = \frac{P_{\text{peak}}}{P_{\text{average}}} = 10 \log_{10} \frac{\max \|x_n\|^2}{E[\|x_n\|^2]}$$  \hspace{1cm} (2-14)$$

Where $P_{\text{peak}}$ represents peak output power, $P_{\text{average}}$ means average output power. $E[\cdot]$ denotes the expected value, $x_n$ represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols $X_k$. Mathematically, $x_n$ is expressed as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k W_N^{nk}$$  \hspace{1cm} (2-16)$$

For an OFDM system with $N$ sub-carriers, the peak power of received signals is $N$ times the average power when phase values are the same. The PAPR of baseband signal will reach its theoretical maximum at $PAPR(dB) = 10\log N$. For example, for a 16 sub-carriers system,
the maximum PAPR is 12 dB. Nevertheless, this is only a theoretical hypothesis. In reality the probability of reaching this maximum is very low.

Fig. 2.11 shows the amplitude characteristic of an OFDM system with 16 sub-carriers. According to the graph, it can be seen that the maximum magnitude of the OFDM signals is less than the upper limit value 16 and corresponding PAPR is also lower than the theoretical maximum 12dB.

![Figure 2.11 An OFDM signal waveform in time domain.](image)

The special case happens when signal sub-carriers are modulated by symbols which have the same initial phase. Assuming that input binary sequence contains ‘1’ for the whole sequence. After PSK constellation mapping and IFFT operation, instant power reaches its theoretical maximum. Fig. 2.12 shows the result when input binary sequence contains 16 ‘1’, denoted by [11111111111111]. In this scenario, the maximum amplitude reaches the value of 16. The PAPR can be calculated from \( PAPR(dB) = 10 \log N \) and in this case it is 12dB.
Figure 2.12 High PAPR when sub-carriers are modulated by same symbols.

By observing the simulation result in Fig. 2.12, we can make a conclusion that the amplitude of OFDM signal reaches its peak value when the input data sequence has a larger consistency. At the same time, the maximum PAPR value will be reached as well.

Another commonly used parameter is the Crest Factor (CF), which is defined as the ratio between maximum amplitude of OFDM signal $s(t)$ and root-mean-square (RMS) of the waveform. The CF is defined as [14]:

$$CF(s(t)) = \frac{\max |s(t)|}{\mathbb{E}[|s(t)|^2]} = \sqrt{\text{PAPR}}$$  \hspace{1cm} (2-17)

In most cases, the peak value of signal $x(t)$ is equals to maximum value of its envelope $|x(t)|$. However, it can be seen from Fig. 2.11 that the appearance of peak amplitude is very rare, thus it does not make sense to use $\max (|x(t)|)$ to represent peak value in real application. Therefore, PAPR performance of OFDM signals is commonly measured by certain characterization constants which are related to probability.
2.4.2 Probability Distribution Function of PAPR

According to central limit theorem, for a large number of sub-carriers in multi-carrier signal, the real and imaginary part of sample values in time-domain will obey Gaussian distribution with mean value of 0 and variance of 0.5. Therefore, the amplitude of multi-carrier signals follows Rayleigh distribution with zero mean and a variance of $N$ times the variance of one complex sinusoid [15]. Its power value obeys a $\chi^2$ distribution with zero mean and 2 degrees of freedom. Cumulative Distribution Function (CDF) is expressed as follows

$$ F(z) = 1 - \exp(-z) $$

(2-18)

Assuming that the sampling values of different sub-channels are mutually independent, and free of oversampling operation, the probability distribution function for PAPR less than a certain threshold value, is therefore expressed as

$$ P(PAPR < z) = F(z)^N = (1 - \exp(-z))^N $$

(2-19)

In practice, it is preferred to take the probability of PAPR exceeding a threshold as measurement index to represent the distribution of PAPR. This can be described as “Complementary Cumulative Distribution Function” (CCDF), and its mathematical expression as

$$ P(PAPR > z) = 1 - P(PAPR \leq z) = 1 - F(z)^N = 1 - (1 - \exp(-z))^N $$

(2-20)

Fig. 2.13 shows the theoretical PAPR’s CCDF distribution with different number of sub-carriers (i.e. $N = 32$, $N = 128$, $N = 1024$). The x-axis represents the PAPR thresholds while the y-axis represents the probability of CCDF. As can be seen from the graph, for a given PAPR threshold, the appearance probability of OFDM symbols which above this threshold PAPR0 will decrease with the increase of sub-carriers number $N$. In this thesis, we will use CCDF to evaluate the performance of various PAPR reduction techniques.
2.4.3 Influencing Factors of PAPR

From the previous sections, it was shown that PAPR is closely related to modulation schemes, number of sub-carriers and oversampling rate.

1. Modulation schemes

Different modulation schemes produce different PAPR performance. Figure 2.14 displays a set of CCDF curves which are processed by several commonly used modulation schemes like BPSK, QPSK, 16QAM and 64QAM with the number of sub-carriers \(N=128\). Results show that there is only small difference between different modulation schemes. Thus, different modulation schemes have minimum influence on PAPR performance.

2. Number of sub-carriers

Different number of sub-carrier results in different PAPR performances due to the varying information carried. When the number of sub-carriers increases, the PAPR also increases.
As shown in Fig. 2.15, when modulation scheme set as QPSK mode, the PAPR exceeds 10 dB accounts for only 0.1% of transmitted OFDM signals when the sub-carrier number is 64, approximately. But when the sub-carrier number rises up to 256, the PAPR exceeds 10 dB accounts for almost 1% of transmitted OFDM signals. Therefore, the number of sub-carrier is a very important influence factor on the PAPR.

3. Oversampling rate

In real implementation, continuous-time OFDM signal cannot be described precisely due to the insufficient $N$ points sampling. Some of the signal peaks may be missed and PAPR reduction performance is unduly accurate [16]. To avoid this problem, oversampling is usually employed, which can be realized by taking $L \cdot N$ point IFFT/FFT of original data with $(L-1) \cdot N$ zero-padding operation. Over-sampling plays an important role for reflecting the variation features of OFDM symbols in time domain. As shown in Fig. 2.16 for a fixed probability, higher over-sampling rate leads to higher PAPR value and good PAPR reduction performance. Generally, oversampling factor $L = 4$ is sufficient to catch the peaks.

Figure 2.14 Comparison of PAPR reduction performances with different modulation schemes.
Figure 2.15 Comparison of PAPR reduction performances with different values of $N$.

Figure 2.16 Comparison of PAPR reduction performances with different values of $L$. 
2.2.4 Studying of PAPR Reduction Techniques

There are many different algorithms that have been proposed to solve the high PAPR problem of OFDM system. These reduction solutions can be roughly divided into three categories:

1. Signal Distortion

One of the most pragmatic and easiest approaches is clipping and filtering which can snip the signal at the transmitter so as to eliminate the appearance of high peaks above a certain level. Clipping can be implemented to the discrete samples prior to digital-to-analog-convertor (DAC) or by designing analog-to-digital-convertor (DAC) and/or amplifier with saturation levels which are lower than the dynamic range [17]. But due to the nonlinear distortion introduced by this process, orthogonality will be destroyed to some extent which results in serious in band noise and out of band noise.

In-band noise cannot be removed by filtering, it decreases the bit error rate (BER). Out-of-band noise reduces the bandwidth efficiency but frequency domain filtering can be employed to minimize the out-of-band power. Although filtering has a good effect on noise suppression, it may cause peak re-growth. To overcome this drawback, the whole process is repeated several times until a desired situation is achieved. Furthermore, some other novel proposals which combine this method with coding and/or signal scrambling have already been studied by other researcher.

2. Signal Scrambling Techniques

The fundamental principle of this technique is to scramble each OFDM signal with different scrambling sequences and select one which has the smallest PAPR value for transmission. Apparently, this technique does not guarantee reduction of PAPR value below to a certain threshold, but it can reduce the appearance probability of high PAPR to a great extent.

This type of approach include: Selective Mapping (SLM) and Partial Transmit Sequences (PTS). SLM method applies scrambling rotation to all sub-carriers independently while PTS method only takes scrambling to part of the sub-carriers. These two methods can be
applied to any scenarios without restriction on the number of sub-carriers and type of modulation. However, for successful recovery of the signal at the receiver, additional information is needed. That leads to low bandwidth utilization and high hardware complexity for implementation.

3. Coding Techniques

The core of encoding method is to apply special forward error correction technique to remove the OFDM signals with high PAPR. The classical schemes include linear block code [18], Golay codes and Reed-Muller code [19]. As far as linear block code method is concerned, it is only suitable to the scenario which has a small number of sub-carriers, which results in limited applications. Reed-Muller code is a high efficiency coding scheme, it obtains a lower PAPR for the second order cosets code by classifying the Walsh-Hadamard transform (WHT) spectrum of the code words. By using Reed-Muller code, PAPR can be reduced to 3dB at most with a good error correcting performance. However, all in all, the encoding method is limited to types of constellation.

A comparison of some promising proposals with respect to comparative factors is given by table 2.1.

**Table 2.1 Comparison of varying PAPR reduction proposals.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Distortion</th>
<th>Power increase</th>
<th>Data rate loss</th>
<th>Operation required at Transmitter (TX) / Receiver (RX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clipping</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>TX: Clipping RX: None</td>
</tr>
<tr>
<td>Coding</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>TX: Coding or table searching RX: Decoding or table searching</td>
</tr>
<tr>
<td>PTS</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>TX: V times IDFTs operation RX: Side information extraction, inverse PTS</td>
</tr>
<tr>
<td>SLM</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>TX: M times IDFTs operation RX: Side information extraction, inverse SLM</td>
</tr>
<tr>
<td>Interleaving</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>TX: D times IDFTs operation, D-1 times interleaving RX: Side information extraction, de-interleaving</td>
</tr>
</tbody>
</table>
Chapter 3

Research on Signal Scrambling Techniques

The emergence of high peak power signal in OFDM system is due to the superposition (IFFT operation) of multiple sub-carrier signals. If multiple sequences which carry the same information are used to represent one transmission process, then the best one can be chosen among those candidates for a given PAPR threshold condition. In this way the occurrence probability of peak power signal can significantly be reduced.

The phase rotation method used for reducing the PAPR of OFDM signal is a special case of multiple signal representation (MSR) technology. Its fundamental principle is: Generating multiple signal waveforms which carry the same information and then choose the waveform from those candidates with the smallest PAPR for transmission. This approach can reduce the occurring probability of high peak power signal effectively, and optimize the statistical characteristics of PAPR in an OFDM system so as to reduce the PAPR successfully. This method is one of the non-distortion methods used for reducing PAPR.

The basic structure of multi-signal representation includes an S/P converter, phase rotation module and parallel output signals which are obtained by executing IFFT operation simultaneously. Finally, the side band information which contains the optimum signal value will be transmitted to the other end of communication. In practice, the side band information can also be encoded by an error-correction code and transmitted through a plurality of reserved sub-carriers [20].

This method is equivalent to performing a linear transformation on modulated data symbols \( X \) in frequency domain. The process can be written as

\[
X_{m,n} = A_{m,n} \cdot X_n \quad (n = 0,1,...,N - 1; m = 1,2,...,M)
\]  

(3-1)

where \( X_n \) represents an element of modulated data symbols \( X \) in frequency domain, \( X_{m,n} \) is the \( N \)-point data symbols before applying IFFT transform. The final goal of this transform is
going to find $N$-point weighting factors $A_{m,n}$. It has the ability to reduce the appearance probability of high peak value $x_n = IFFT(X_{m,n})$ in time domain. Block diagram of phase rotation PAPR reduction scheme as shown in Fig. 3.1.

![Block diagram of phase rotation](image)

Figure 3.1 Block diagram of phase rotation.

This method has lots of merits, such as high coding rate and low redundancy, although it only optimizes the statistical characteristics of PAPR in OFDM system. Therefore, varying schemes based on this principle have a bright application prospect.

Phase rotation method contains a lot of different schemes. However, until now, there exist two most effective and meritorious proposals which are called SLM and PTS. In this thesis, an investigation of these two potent probabilistic approaches is conducted. The two have received so much attention since they both can provide a low reduction in throughput and have relatively high bandwidth utilization.

### 3.1 Selected Mapping Method

The CCDF of the original signal sequence’s PAPR above a threshold PAPR0 is written as $Pr\{PAPR > PAPR_0\}$. Thus for $K$ statistical independent signal waveforms, CCDF can be rewritten as $[Pr\{PAPR > PAPR_0\}]^K$, so that the probability of PAPR that exceeds the same threshold will drop to a small value.
3.1.1 Principle of SLM (Selected Mapping)

The probability of PAPR larger than a threshold $z$ can be written as $P(PAPR > z) = 1 - (1 - \exp(-z))^N$. Assuming that $M$ OFDM symbols carry the same information and that they are statistically independent of each other. In this case, the probability of PAPR greater than $z$ is equals to the product of each independent candidate’s probability. This process can be written as

$$
P(P_{APR_{low}} > z) = (P(PAPR > Z))^M = ((1 - \exp(-z))^N)^M
$$

In selected mapping method, firstly $M$ statistically independent sequences which represent the same information are generated, and next, the resulting $M$ statistically independent data blocks $S_m = [S_{m,0}, S_{m,1}, ..., S_{m,N-1}]^T$, $m = 1, 2, ..., M$ are then forwarded into IFFT operation simultaneously. Finally, at the receiving end, OFDM symbols $x_m = [x_1, x_2, ..., x_N]^T$ in discrete time-domain are acquired, and then the PAPR of these $M$ vectors are calculated separately. Eventually, the sequences $x_d$ with the smallest PAPR will be elected for final serial transmission. Fig. 3.2 illustrates the basic structure of selected mapping method for suppressing the high PAPR.

![Figure 3.2 Basic principles of selected mapping.](image)

This method can significantly improve the PAPR performance of OFDM system. The reasons behind that are: Data blocks $S_m = [S_{m,0}, S_{m,1}, ..., S_{m,N-1}]^T$, $m = 1, 2, ..., M$ are statistical independent, assuming that for a single OFDM symbol, the CCDF probability of PAPR larger
than a threshold is equals to \(p\). The general probability of PAPR larger than a threshold for \(k\) OFDM symbols can be expressed as \(p^K\). It can be verified that the new probability obtained by SLM algorithm is much smaller compared to the former. Data blocks \(S_m\) are obtained by multiplying the original sequence with \(M\) uncorrelated sequence \(P_m\).

Fig. 3.3 shows the theoretical CCDF curves as a function of PAPR distribution when SLM method is used. The number of \(N\) sub-carriers is 128. \(M\) takes the value of 1 (without adopting SLM method), 2, 8, 32 and 128. It is seen in Fig. 3.3 that with increase of branch number \(M\), PAPR’s CCDF distribution gets smaller and smaller.

![Theoretical PAPR’s CCDF curves using SLM method.](image)

Figure 3.3 Theoretical PAPR’s CCDF curves using SLM method.

The key point of selected mapping method lies in how to generate multiple OFDM signals when the information is the same. First, defined different pseudo-random sequences \(P_m = [P_{m,0}, P_{m,1}, ..., P_{m,N-1}]^T\), \(m = 1, 2, ..., M\), where \(P_{m,n} = e^{j\phi_{m,n}}\) and stands for the rotation factor. \(P_{m,n}\) is also known as the weighting factor. \(\phi_{m,n}\) is uniformly distributed in \([0, 2\pi]\). The \(N\) different sub-carriers are modulated with these vectors respectively so as to generate candidate OFDM signals. This process can also be seen as performing dot product operation on a data block \(X_n\) with rotation factor \(P_m\).
In the reality, all the elements of phase sequence $P_1$ are set to 1 so as to make this branch sequence the original signal. The symbols in branch $m$ is expressed as

$$S_m = [X_0P_{m,0},X_1P_{m,1},...,X_{N-1}P_{m,N-1}]^T, \ m = 1,2,...,M$$

and then transfer these $M$ OFDM frames from frequency domain to time domain by performing IFFT calculation. The entire process is given by

$$x_m(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n P_{m,n} \cdot e^{j2\pi n\Delta f t}, \ 0 \leq y \leq NT, m = 1,2,...M$$

Finally, the one which possess the smallest PAPR value is selected for transmission. Its mathematical expression is given as

$$x_d = \arg \min_{1 \leq m \leq M}(PAPR(x_m))$$

where argmin(·) represent the argument of its value is minimized.

At the receiver, in order to correctly demodulate the received signal, it is necessary to know which sequence is linked to the smallest PAPR among $M$ different candidates after performing the dot product. Hence, the receiver is required to learn information about selected phase vector sequence and ensure that the vector sequence is received correctly.

An intuitive approach is to select the whole sequence of branch number $m$ as side information transmitted to the receiving end. However, in practice, the process does not necessarily require the delivery of the entire vector sequence. It can be realized by sending the route number of the vector sequence instead. This is only possible when the receiving end is able to restore the random phase sequence $P_m$ by means of look-up table or any other method.

Since the side information plays a vital role for signal restoration at the receiver, channel coding is used to guarantee a reliable transmission. Once channel coding technique is adopted during the data transmission process, sending of any additional side information is not required. In this way, all possible routes are detected at the receiving end from which the most likely one is chosen as the optimum. Considering that the emphasis of this chapter is to explore the principle of SLM algorithm and evaluate different related factors, the recovery process of original sequence will not be discussed in detail.
3.1.2 Simulation of SLM Scheme

In this part, an evaluation of factors which could influence the PAPR reduction performance is performed using Matlab simulation. Based on the principles of SLM algorithm, it is apparently that the ability of PAPR reduction using SLM is affected by the route number $M$ and subcarrier number $N$. Therefore, simulation with different values of $M$ and $N$ will be conducted, and the results exhibits some desired properties of signals representing the same information [21].

1. Comparison of PAPR reduction performance with different values of $M$ while $N$ is fixed at 128

Firstly from the perspectives of complexity and practicability, rotation factor is defined as $P_{m,n} \in [\pm 1, \pm j]$. This reduces calculation complexity dramatically compared to performing miscellaneous complex multiplication. The algorithm executes 10000 times, over-sampling factor is 8 and QPSK mapping is adopted as modulation scheme in each sub-carrier. Route numbers $M=2$, $M=4$, $M=8$, $M=16$ and $M=32$ are used. From Fig. 3.4, it can be observed that the proposed SLM method displays a better PAPR reduction performance than the original OFDM signal which is free of any PAPR reduction scheme.

The probability of high PAPR is significantly decreased. Increasing $M$ leads to the improvement of PAPR reduction performance. If the probability is set to 1% and then the CCDF curves with different $M$ values are compared. The PAPR value of case $M=2$ is about 1dB smaller than the unmodified one $M=1$. Under the same condition, the PAPR value of case $M=16$ is about 3dB smaller than the original one $M=1$. However, from the comparison of the curve $M=8$ and $M=16$, we learned that the performance difference between these two cases is less than 0.5dB. This proves that we will not be able to achieve a linear growth of PAPR reduction performance with further increase the value of $M$ (like $M>=8$), the PAPR reduction performance of OFDM signal will not be considerably improved.

Furthermore, from the perspective of execution time, we can see that execution time will last longer with the increase of $M$. Therefore, in practical application, we usually take $M=8$, thereby not only improve the system performance, but also avoid introducing too
much computational complexity so as to save the limited resource successfully.

2. Comparison of PAPR reduction performance with different $N$ values while $M$ is fixed at 8

In this case, we set the number of OFDM signal frame $M$ equals to 8, the number of sub-carrier $N$ equals to 256, 128, and 64, respectively. In the Fig. 3.5, the CCDF curve of original sequence’s PAPR is given as the reference of comparison to the others which SLM method been used. It can be seen from Fig. 3.5 that SLM algorithm particularly suitable for the OFDM scenario with larger number of sub-carriers, and it also shows the PAPR reduction performance of OFDM signal is not significantly deteriorated when the number of sub-carriers is greater than 128, even if the number of carriers doubled after the adoption of SLM algorithm.

![Figure 3.4 Comparison of PAPR reduction performances with different values of $M$.](image)
Figure 3.5 Comparison of PAPR reduction performances with different values of $N$.

The following conclusions can be drawn after the analysis and comparison of above groups of simulation results:

1. SLM proposal can significantly improve the PAPR distribution of OFDM system, that is, significantly reduce the presenting probability of large peak power signal. The increasing of the number of OFDM signal frames $M$ will raise the complexity dramatically, but with benefit of small improvement of PAPR reduction performance.

2. SLM algorithm adapted to any length of FFT frame that means it can be used for different OFDM systems with different number of carriers. It is particularly suitable for the OFDM system with a large number of sub-carriers (more than 128).

3. SLM can significantly improve the performance of OFDM system by reducing the PAPR, but at the same time, the price is also very clear that is the complexity of its implementation. Every time when applying SLM algorithm, requires calculating the $M$ group IFFTs at the transmitter compared to only one on ordinary OFDM system, and its $M$
of $N$ points IFFTs operation needs $n_{mul} = M \cdot \frac{N}{2} \log_2 N$ complex multiplication and

$$n_{add} = M \cdot \frac{N}{2} \log_2 N$$

addition, separately.

These problems pose a very heavy burden on real OFDM implementation; we need to reduce the computational complexity. Therefore, in practical application, compromise the computing complexity and improvement of performance, we usually take $M \leq 8$.

### 3.2 Partial Transmit Sequence

#### 3.2.1 Principle of PTS (Partial Transmit Sequence)

Partial Transmit Sequence (PTS) algorithm was first proposed by Müller S H, Huber J B [21][22], which is a technique for improving the statistics of a multi-carrier signal. The basic idea of partial transmit sequences algorithm is to divide the original OFDM sequence into several sub-sequences, and for each sub-sequence, multiplied by different weights until an optimum value is chosen.

![Block diagram of PTS algorithm](image)

**Figure 3.6 Block diagram of PTS algorithm.**

Fig. 3.6 is the block diagram of PTS algorithm. From the left side of diagram, we see that the data information in frequency domain $X$ is separated into $V$ non-overlapping sub-blocks and each sub-block vectors has the same size $N$. Hence, we know that for every sub-block, it
contains $N/V$ nonzero elements and set the rest part to zero. Assume that these sub-blocks have the same size and no gap between each other, the sub-block vector is given by

$$
\mathbf{X} = \sum_{v=1}^{V} b_v \mathbf{X}_v
$$

(3-6)

where $b_v = e^{j\psi_v} (\psi_v \in [0,2\pi]) \{v = 1,2, ..., V\}$ is a weighting factor been used for phase rotation. The signal in time domain is obtained by applying IFFT operation on $X_v$, that is

$$
\hat{x} = \text{IFFT}(\mathbf{X}) = \sum_{v=1}^{V} b_v \text{IFFT}(\mathbf{X}_v) = \sum_{v=1}^{V} b_v \cdot \mathbf{x}_v
$$

(3-7)

Select one suitable factor combination $\mathbf{b} = [b_1, b_2, ..., b_v]$ which makes the result achieve optimum. The combination can be given by

$$
\mathbf{b} = [b_1, b_2, ..., b_v] = \arg \min_{(b_1,b_2,...,b_v)} (\max_{1 \leq n \leq N} |\sum_{v=1}^{V} b_v x_v|^2)
$$

(3-8)

where $\arg \min (\cdot)$ is the judgment condition that output the minimum value of function. In this way we can find the best $\mathbf{b}$ so as to optimize the PAPR performance. The additional cost we have to pay is the extra $V-1$ times IFFTs operation.

In conventional PTS approach, it requires the PAPR value to be calculated at each step of the optimization algorithm, which will introduce tremendous trials to achieve the optimum value [21]. Furthermore, in order to enable the receiver to identify different phases, phase factor $\mathbf{b}$ is required to send to the receiver as sideband information (usually the first sub-block $b_1$, is set to 1). So the redundancy bits account for $(V - 1) \log_2 W$, in which $V$ represents the number of sub-block, $W$ indicates possible variations of the phase. This causes a huge burden for OFDM system, so studying on how to reduce the computational complexity of PTS has drawn more attentions, nowadays.

The optimization is achieved by searching thoroughly for the best phase factor. Theoretically, $\mathbf{b} = [b_1, b_2, ..., b_v]$ is a set of discrete values, and numerous computation will be required for the system when this phase collection is very large. For example, if $\psi_v$ contains $W$ possible values, theoretically, $\mathbf{b}$ will have $W^V$ different combinations, therefore, a total of $V \cdot W^V$ IFFTs will be introduced.

By increasing the $V$, $W$, the computational cost of PTS algorithm will increase exponentially. For instance, define phase factor $b_v$ contains only four possible values, that means $b_v \in \{1,2,3,4\}$, the combination of $\mathbf{b}$ can be expressed as $\{1,2,3,4\}^V$, and the total number of possible combinations is $4^V$. The additional cost of this method is $V$ times IFFTs operation.
$[\pm 1, \pm j]$, then for each OFDM symbol, $2 \cdot (V - 1)$ bits are transmitted as side information. Therefore, in practical applications, computation burden can be reduced by limiting the value range of phase factor $\mathbf{b} = [b_1, b_2, ..., b_v]$ to a proper level. At the same time, it can also be changed by different sub-block partition schemes.

### 3.2.2 Modified PTS Scheme

Based on the discussion above, we had realized that the PAPR reduction performance and computational complexity of PTS algorithm is closely related to the sub-block partition and the value range of weighting factor. Thus, in practical, an effective compromise between system complexity and system performance should be made by choosing appropriate sub-block partition scheme and simpler searching scheme for optimum weighting factor. Meanwhile, in traditional PTS reduction scheme, the optimal weighting factor is calculated by thoroughly searched phase factors in a finite set $[23]$, which will increase the complexity of real application.

From this point of view, a suboptimal algorithm is going to be introduced in this section, which is able to achieve a satisfied performance with minimal number of trials.

1. Sub-block partition scheme

In PTS-OFDM system, there exist three seed sub-block partition schemes: adjacent partition, pseudo-random partition and interleaved partition.

In Fig. 3.7, graphs are shown for the illustration of these three partition schemes. From the figure we can see that adjacent partition is divide sequence into $V$ sub-blocks, for each one, it contains $N/V$ consecutive sub-carriers; In pseudo-random partition, each sub-carrier can be randomly assigned to any position of sub-block with the length $V$; Interleaved partition also segments the sequence into $V$ sub-blocks but within each of them, sub-carriers are allocated in a space of $V$. The common point of these three different partition schemes is that each sub-carrier is only been assigned once, and the length of each sub-sequence is same.
Figure 3.7 Illustration of different sub-block partition schemes: (a) adjacent partition, (b) pseudo-random partition, (c) interleaved partition.
For $M$ statistically independent OFDM signals, the CCDF has been given by Eq. (3-2). However, it is comprehensible that these candidates are not strictly mutual independent since they all represent the same information of original data. Actually, Eq. (3-2) only indicates the theoretical boundary in a PTS-OFDM system. Besides, theory also proves that the less correlated of candidates are, the better PAPR reduction performance of an OFDM system will be [24]. It means that we need to reduce the correlation so as to approach the CCDF theoretical boundary as much as possible, and for PTS method, sub-blocks partition is related to correlation property directly.

After analyzing the auto correlation function of these three different sub-block partition schemes, we know that for pseudo random sub-block partition, we can derive [25]

$$
|R_{xy}(\tau)| = \frac{1}{N} |\sum_{\nu=1}^{\nu} b_x^\nu b_y^{\nu^*}| \cdot |\sum_{k \in \phi} e^{j2\pi k\tau N}| (3-9)
$$

If $\tau \neq 0$, when $V$ is small, since the allocation of sub-carriers is random in any sub-block, the value of Eq. (3-9) is approximates to 0. This can rewrite as

$$
|R_{xy}(\tau)| \approx \begin{cases} 
0 & , \tau \neq 0 \\
\frac{1}{V} |\sum_{\nu=1}^{\nu} b_x^\nu b_y^{\nu^*}| & , \tau = 0
\end{cases} (3-10)
$$

From the formula we notice that the strongest correlation value only exists at $\tau = 0$, at other time shift points, the values of auto correlation are very small. Hence, the pseudo random sub-block partition scheme has a unique merit on this point.

2. Suboptimal iterative algorithm

As discussed above, the traditional PTS method is a traversal algorithm, which requires all possible phase values to be evaluated, that actually constrain the real application. For this reason, a suboptimal algorithm is going to be introduced to reduce the number of phase patterns. It can reduce the complexity effectively and combine the advantage of PTS scheme on PAPR reduction performance, but only has small degree of degradation compared to the optimum, although the solution retrieved from this algorithm is not the optimal solution [22].
The detailed steps are shown as follows:

1) Divide $N$ sub-carriers into $V$ non-overlapping sub-blocks.

2) Assume that $\mathbf{b} = [b_1, b_2, ..., b_v] = [1,1, ...,1]$, and then calculate the PAPR value of an intermediate sequence $x'$ in time domain, write it as $\text{PAR}_{x'}$. Initialize $\text{index} = 1$, which represents the subscript of $b_{\text{index}}$.

3) Assume that $b_{\text{index}} = -1$, and calculate the PAPR of this new sequence $\hat{x}$, write it as $\text{PAR}_{\hat{x}}$.

4) If $\text{PAR}_{\hat{x}} > \text{PAR}_{x'}$, let $b_{\text{index}} = -1$, otherwise, $P_{x'} = P_{\hat{x}}$, and make $\text{index} = \text{index} + 1$.

5) If $\text{index} < V + 1$, go to step 3), otherwise jump to step 6). The iteration continues until $\text{index} = V$.

6) Obtain sub-optimal weighting factor $\mathbf{b}$ and corresponding PAPR is $\min (P_{\hat{x}}, P_{x'})$.

Adopting iterative algorithm for searching suboptimal weighting factor $\tilde{\mathbf{b}}$, only $V$-steps calculation are needed. In each step, the IFFT calculation only be performed for corresponding one sub-block rather than calculate the all $V$ sub-blocks, by this means, it can reduce the complexity of the calculation significantly. The flow chart of suboptimal algorithm is shown in Fig. 3.8.
Figure 3.8 Flowchart of suboptimal iterative algorithm.

\[ b = [b_1, b_2, ..., b_v] = 1 \text{, then calculate time domain sequence } x \text{ and its PAPR value } PAR_x. \text{ Define index } = 1 \]

Set \( b_{\text{index}} = -1 \), and then calculate the new sequence \( \hat{x} \) and its PAPR value \( PAR_{\hat{x}} \)

- If \( PAR_{\hat{x}} > PAR_x \), then
  - \( PAR_x' = PAR_{\hat{x}} \)
  - \( \text{index} = \text{index} + 1 \)

- If \( \text{index} < v + 1 \), then
  - Obtain the optimized weighting factor \( b \)

End
3.2.3 Simulation of PTS Scheme

We realized from the above discussion that in PTS approach, there are varying parameters impact the PAPR reduction performance, these are: 1) The number of sub-blocks $V$, which influences the complexity strongly; 2) The number of possible phase value $W$, which impacts the complexity as well; and 3) The sub-block partition schemes. In our simulation, two parameters will be considered. They are sub-block sizes $V$ and different sub-block partition proposals.

1. PAPR reduction performance effects by number of sub-blocks $V$

Simulation evaluates the PAPR reduction performance using PTS algorithm with different $V$, in which simulation configuration, QPSK is applied, $N = 256$ and $V = 0, 2, 3, 4$, respectively.

From Fig. 3.9, it can be seen that PTS algorithm undeniably improve the performance of OFDM system, moreover, with the increasing of $V$, the improvement of PAPR reduction performance becomes better and better. Assume that we fix the probability of PAPR at 1%, and compare the CCDF curve with different $V$ values. Form the figure, we notice that the CCDF curve has nearly 1.5dB improvement when $V = 2$, compared to the conventional OFDM system. When $V = 4$, the 1% PAPR is about 6.6dB, so an optimization of more than 3 dB is achieved.

However, the downward trend of CCDF curve is tended to be slow when we keep on increasing $V$, which means too large sub-block numbers $V$ will result in small improvement of PAPR reduction performance, but pay for the tremendous hardware complexity. Therefore, practically, we prefer to choose a suitable value of $V$ to achieve a tradeoff in the use of PTS.

2. PAPR reduction performance effects by different value range $W$

The simulation result in Fig. 3.10 shows the varying PAPR reduction performance with different $W$ (collection range of weighting factor $b_v$) when using PTS reduction scheme. Simulation specific parameters are: the number of sub-carriers $N = 128$, QPSK
constellation modulation, oversampling factor takes $L = 8$, the number of sub-block $V = 4$. From the figure we notice that the CCDF curve has nearly 1dB improvement when $W = 4$, compared to $W = 2$, the 1% PAPR is about 7.5 dB. We conclude that in a PTS-OFDM system, the larger $W$ value takes, the better PAPR performance will be obtained when the number of sub-block $V$ is fixed.

3. PAPR reduction performance effects by different sub-block partition schemes

Fig. 3.11 displays the PAPR reduction performance using PTS proposal with different sub-block partition schemes, in which QPSK is applied, $N = 256$ and $V = 0, 2, 3, 4$, respectively. As shown in Fig. 3.11, the CCDF curve which is represented by dotted line using adjacent partition scheme, and solid line is plotted based on the pseudo-random partition scheme. The CCDF curves from left to right correspond to the case with sub-block number $V = 4$, $V = 3$ and $V = 2$ respectively. As we can see from the graph that with the same $V$ been taken, system’s performance of pseudo-random partition is superior to the one based on adjacent partition.

4. Comparison of iterative (sub-optimization) algorithm and optimization algorithm

For PTS-OFDM system, simulation is carried out with the number of sub-carriers $N=128$, and applying QPSK modulation, pseudo-random partition scheme is adopted for these two algorithms, and sub-block number takes $V = 4$. For the convenient of comparison, the range of weighting factor sets to two for both cases, which is $b_v = \left[ \pm 1 \right]$. Simulation result is shown in Fig. 3.12 and some conclusions reached on the basis of observation: the CCDF curve of sub-optimization PTS algorithm lies in the middle of unmodified system’s CCDF curve and CCDF curve processed by conventional PTS method, in another word, as might be expected, sub-optimization algorithm does not provide the best performance due to its inherent disadvantages. However, in real application, since sub-optimal algorithm can reduce system complexity greatly, we prefer to choose it instead of the conventional optimum algorithm.
Figure 3.9 Comparison of PAPR reduction performances with different values of $V$.

Figure 3.10 Comparison of PAPR reduction performances with different values of $W$. 

46
Figure 3.11 Comparison of PAPR reduction performances with difference sub-block partition schemes.

Figure 3.12 Comparison of PAPR reduction performances between iterative (sub-optimization) algorithm and optimization algorithm for PTS-OFDM system.
3.2.4 Comparison of SLM and PTS Algorithm

1. Comparison of auxiliary information

SLM and PTS algorithms are two typical non-distortion techniques for reducing PAPR in OFDM system. In order to have error-free demodulation in the receiving end, side information must be sent to the receiver, correctly. Hence, in practical application often requires the use of some coding measures to protect information from being disturbed. Since this thesis only focuses on studying PAPR reduction performance in OFDM system with different algorithms, and does not reflect on the modulation in receiving end. Thus, we will look at the redundancy of auxiliary information rather than coding redundancy.

In PTS method, if the collection range of weighting factor is \( W \), then for \( V \) sub-blocks, the system exists \( W^{V-1} \) types of auxiliary information sequence, so the number of redundant bits is \( R_{ap} = (V - 1) \log_2 W \). By the same token, in SLM method, if the length of sequence \( P_m \) is \( M \), then in SLM-OFDM system, it requires redundant bits \( R_{ap} = \log_2 (M - 1) \). As can be seen from Table 3.1, under the same circumstances, PTS method requires a higher information redundancy, compare to SLM algorithm.

Table 3.1 The number of redundant bits used in PTS and SLM method with different \( V \) and \( M \)

<table>
<thead>
<tr>
<th>Scrambling technique</th>
<th>Partition number</th>
<th>( V=M=2 )</th>
<th>( V=M=4 )</th>
<th>( V=M=8 )</th>
<th>( V=M=16 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTS</td>
<td>( W=2 )</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>( W=4 )</td>
<td>2</td>
<td>6</td>
<td>14</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>( W=8 )</td>
<td>3</td>
<td>9</td>
<td>21</td>
<td>45</td>
</tr>
<tr>
<td>SLM</td>
<td>0</td>
<td>1.58</td>
<td>2.81</td>
<td>3.91</td>
<td></td>
</tr>
</tbody>
</table>

2. Comparison of PAPR reduction performances between SLM and PTS method

Fig. 3.13 shows the simulation result of using SLM and PTS method to an OFDM system, separately. In PTS method, we set the number of sub-carriers \( N = 128 \) and applying pseudo-random partition scheme, for each carrier, adopting QPSK constellation mapping,
weighting factor \( b_v \in [\pm 1, \pm j] \); In SLM method, rotation factor \( P_{m,n} \in [\pm 1, \pm j] \). Based on the theory, we know that the IFFT calculation amount of these two methods is same when \( V = M \), but for PTS method, it can provide more signal manifestations, thus, PTS method should provide a superior performance on PAPR reduction. In fact, this deduction is confirmed by simulation result. From the Figure 3.13, we learned that with the same CCDF probability 1%, the PAPR value equals to 7dB when PTS is employed, while the PAPR raise up to 8.2dB when SLM is employed under the same circumstance.

Figure 3.13 Comparison of PAPR reduction performances between PTS algorithm and SLM algorithm for PTS-OFDM system.

It shows clearly that PTS method provides a better PAPR reduction performance compared to SLM method. Nevertheless, the cost is also payed for sacrificing transmission efficiency and rising complexity. Thus, in practical applications, a tradeoff should be made between good performance and auxiliary information. From the discussion above, we can say that SLM algorithm is more suitable if system can tolerate more redundant information, otherwise, PTS algorithm is more acceptable when complexity becomes the first considering factor. In brief, compromise will be made for a reliable system.
Chapter 4

Conclusion and Future Work

4.1 Conclusion

First of all, this thesis provides an overview of Multiple-Input-Multiple-Output (MIMO) technology and Orthogonal-Frequency-Division-Multiplexing (OFDM). At the same time, the advantages and disadvantages of OFDM system are concluded by analyzing and comparing it with other traditional modulation schemes. The focus of thesis is that we investigate one of the bottleneck problems that exist in OFDM wireless communication system – high peak-average power ratio (PAPR) of OFDM signal, and discuss how to reduce it by different effective algorithms. The main contributions of this thesis are listed below:

1. The comprehensive research and comparison are put forward for a variety of currently promising PAPR reduction methods on the basis of extensive reading and studying of associated paper and literature in this research area. Among these different proposals, we are mainly focusing on the signal scrambling technology, and verify the theoretical analysis by observing the MATLAB simulation results. At the same time, some meaningful guidance and conclusions are obtained through the comparative analysis of these simulation results as well.

2. In the signal scrambling technology, we study the method of selected mapping and partial transmit sequence. A series of detailed comparison results were obtained of these two schemes from the angle of PAPR reduction performance, redundancy of auxiliary information, as well as complexity of system. Summed up the advantages and disadvantages of two algorithms and pointed out that the occasions of their respective adaptation. For the inherent defect of traditional PTS algorithm – complex computing, a very effective iterative method is introduced to determine sub-optimal weighting factor for each sub-block instead of conducting an ergodic searching so as to reduce the calculation complexity significantly. This sub-optimal algorithm gives a better approach
to the real conditions in engineering practice by providing a compromise between the
PAPR reduction performance and computational complexity. At last, we also compare
these two schemes under the same conditions in general, and some conclusions were
drawn which are constructive to practical work.

4.2 Suggestion for Future Work

OFDM as a multi-carrier modulation technique particularly suited for high-speed wireless
transmission. Our studying is mainly focused on the evaluating of various PAPR reduction
performances in OFDM system. However, there are still many technical problems to be
resolved although its excellent characteristics manifested in almost all aspects of wireless
communications.

1. In this thesis, all the simulation results are acquired under ideal conditions, but in reality,
OFDM system has lots of practical problems, such as synchronization, channel estimation.
Hence, for establishing a more complete and credible simulation system, synchronization
and channel estimation techniques can be added to OFDM system simulation platform.
Besides, all these PAPR reduction methods can be applied to MIMO-OFDM system.

2. Through the actual simulation process, we realize that the simulation routines used in our
thesis is time consuming and results are fallible due to the limitation of Matlab simulation
system. In subsequent researching, simulation software - Visual C (VC) can be used to
simulate the communication rate, or make it become a professional module of System
View communication simulation tool so as to improve the simulation accuracy and
efficiency. In addition, the application of DSP processor can be considered to complete
the simulation of real-time data. That means, to build a hardware OFDM simulation
platform based on DSP processor, which will provide more solid theoretical basis and
guidance for the practical application of OFDM system.

3. Due to the characteristics of multi-antenna of MIMO-OFDM system itself, we can fully
explore the advantages of combination between proposed PAPR reduction schemes and
outstanding properties of MIMO-OFDM system, such as studying the PAPR reduction
technology of MIMO-OFDM system, combine with space-time codes.
Reference


Appendix A

1. Flow chart of routine used in Figure 3.4 (Comparison of SLM PAPR reduction performance with different values of $M$ while $N$ is fixed at 128).

![Flow chart of routine used in Figure 3.4](image)

2. Flow chart of routine used in Figure 3.10 (Comparison of PTS PAPR reduction performance effects by different value range $W$).

![Flow chart of routine used in Figure 3.10](image)
3. Flow chart of routine used in Figure 3.13 (Comparison of PAPR reduction performances between SLM and PTS method).

Start

Initialize related parameters (Branch numbers K in SLM method, sub-block numbers V in PTS method, etc.)

Generate OFDM symbols with QPSK modulation for each sub-carriers and set weighting factor used for SLM and PTS method both

Generate all possible combinations of weighting factor set in PTS method

Applying SLM and PTS algorithm separately with the number of generated OFDM symbols equals to 1e3

Calculate corresponding PAPR Empirical Cumulative Distribution Function (ECDF) values

Calculate and plot Complementary Cumulative Distribution Function (CCDF) of different PAPR

End
Appendix B

```matlab
% SLM_vari_M_Figure3.4 %
%******************************************************************************%
%NAME:  Wang Yi              P-NUMBER 8311048659
%NAME:  Gu Linfeng          P-NUMBER 8411125118
%******************************************************************************
clc;
close all;
clear all;

N = 128; % The number of carriers
OF = 8;  % Oversampling factor
K = N*OF;
QPSK_Set = [1 -1 j -j]; % QPSK constellation symbols
Phase_Set = [1 -1 j -j]; % Define the rotation factor value range
Max_Symbols = 1e3; % The number of generated OFDM symbols

hwait = waitbar(0,'Please wait...'); % Creates and displays a waitbar

for nSymbol = 1: Max_Symbols
    Index = randint(1,N,length(QPSK_Set))+1; % Generate the random QPSK constellation indexes
    X = QPSK_Set(Index(1,:)); % OFDM symbol in frequency domain after QPSK constellation mapping
    X2 = [X(1:N/2) zeros(1,K-N) X(N/2+1:N)]; % Oversampling process
    x = ifft(X2,[],2); % Signals in time domain after IFFT operation
    Signal_Power = abs(x.^2);
    Peak_Power = max(Signal_Power,[],2);
    Mean_Power = mean(Signal_Power,2);
    PAPR_temp = 10*log10(Peak_Power/Mean_Power);
    PAPR_Orignal(nSymbol) = PAPR_temp; % The PAPR value of original OFDM signals
end

[cdf1, PAPR1] = ecdf(PAPR_Orignal); % Calculate PAPR's empirical cumulative distribution of original signal

M = [2 4 8 16]; % Define different route numbers used in SLM method
PAPR_SLM=zeros(4,64);

for m=1:4
    X = zeros(M(m),N);
    Index = zeros(M(m),N);
    step = Max_Symbols /100; % Set the parameters of
    waitbar
    for nSymbol = 1: Max_Symbols
        if Max_Symbols-nSymbol<=50
```

57
waitbar(nSymbol/ Max_Symbols,hwait,'Almost done!');
pause(0.05);
else
    PerStr=fix(nSymbol/step);
    str=['Process on going>>>',num2str(PerStr),'%'];
    waitbar(nSymbol/ Max_Symbols,hwait,str);
    pause(0.05);
end

Index(1,:) = randint(1,N,length(QPSK_Set))+1;
Index(2:M(m),:) = randint(M(m)-1,N,length(Phase_Set))+1; % Generate the random rotation factor indexes

X(1,:) = QPSK_Set(Index(1,:)); % Symbols in frequency domain after Modulation
Phase_Rot = Phase_Set(Index(2:M(m),:)); % Corresponding rotation factor Pm were created
X(2:M(m),:) = repelem(X(1,:),M(m)-1,1).*Phase_Rot; % Multiple OFDM matrix (D(m)*N) with rotation factors
X2 = [X(:,1:N/2) zeros(M(m),K-N) X(:,N/2+1:N)]; % Oversampling process
x = ifft(X2,[],2); % Signals in time domain after IFFT operation

Signal_Power = abs(x.^2);
Peak_Power = max(Signal_Power,[],2);
Mean_Power = mean(Signal_Power,2);
PAPR_temp = 10*log10(Peak_Power./Mean_Power);
PAPR_SLM(m,nSymbol)= min(PAPR_temp); % The PAPR values of OFDM signals using SLM method with different M

end
end

close(hwait);

[cdf2, PAPR2] = ecdf(PAPR_SLM(1,:)); % Calculate signal's PAPR empirical cumulative distribution(SLM PAPR reduction method employed with M=2)

[cdf3, PAPR3] = ecdf(PAPR_SLM(2,:)); % Calculate signal's PAPR empirical cumulative distribution(SLM PAPR reduction method employed with M=4)
[cdf4, PAPR4] = ecdf(PAPR_SLM(3,:)); % Calculate signal's PAPR empirical cumulative distribution (SLM PAPR reduction method employed with M=8)

[cdf5, PAPR5] = ecdf(PAPR_SLM(4,:)); % Calculate signal's PAPR empirical cumulative distribution (SLM PAPR reduction method employed with M=16)

semilogy(PAPR1,1-cdf1,'linewidth',2) % Plot CCDF curves of PAPR with different M value

hold on;
semilogy(PAPR2,1-cdf2,'linewidth',2) hold on;
semilogy(PAPR3,1-cdf3,'linewidth',2) hold on;
semilogy(PAPR4,1-cdf4,'linewidth',2) hold on;
semilogy(PAPR5,1-cdf5,'linewidth',2);
axis([5 12 10e-4 1])
grid on;
legend('Original',' M=2',' M=4',' M=8',' M=16');
xlabel('PAPR0 [dB]');
ylabel('CCDF (Pr[PAPR>PAPR0])');
clc;
clear all;
close all;

K = 128;  % The number of carriers
QPSK_Set = [1 -1 j -j];  % QPSK Constellation symbols
Phase_Set1 = [1 -1];  % Weighting factor set 1
Phase_Set2 = [1 -1 j -j];  % Weighting factor set 2
V = 4;  % The number of sub-blocks

Choose1 = [1 1 1 1; 1 1 2 1; 1 2 1 1; 2 1 1 1;...
1 1 2 2; 1 2 2 1; 2 2 1 1; 2 1 2 1; 2 1 1 2;...
2 2 2 1; 2 2 1 2; 2 1 2 2; 1 2 2 2; 2 2 2 2];
% Generate matrix of all possible combinations with weighting factor set 1
Choose_Len1 = 16;  % The total number of combinations or IFFT operations using weighting factor set 1
hwait = waitbar(0,'Please wait...');  % Creates and displays a waitbar

for i=1:4  % Generate all possible combinations of 4 branches with weighting factor set 2
X(i,1:4^i) = [repmat(1,1,4^(i-1)),repmat(2,1,4^(i-1)),repmat(3,1,4^(i-1)),repmat(4,1,4^(i-1))];

Y = X(i,1:256);
X(i,1:256) = repmat(Y,1,256/length(Y));
End

X = X.';
Choose2 = fliplr(X);
Choose_Len2 = 256;  % The total number of combinations or IFFT operations using weighting factor set 2

Max_Symbols = 1e3;  % The number of generated OFDM symbols
PAPR_Original = zeros(1,Max_Symbols);
PAPR_PTS = zeros(1,Max_Symbols);

for nSymbol = 1:Max_Symbols *10
Index = randint(1,K,length(QPSK_Set))+1;  % Generate the random QPSK constellation indexes

X = QPSK_Set(Index(1,:));  % The QPSK modulation

x = ifft(X,[],2);  % Signals in time domain after IFFT operation
Signal_Power = abs(x.^2);
Peak_Power = max(Signal_Power,[],2);
Mean_Power = mean(Signal_Power,2);
PAPR_Original(nSymbol) = 10*log10(Peak_Power./Mean_Power);  % The PAPR value of original
OFDM signals

step = Max_Symbols /100; % Set the parameters of waitbar
for nSymbol = 1: Max_Symbols

    if Max_Symbols-nSymbol<=50
        waitbar(nSymbol/ Max_Symbols,hwait,'Almost done!');
        pause(0.05);
    else
        PerStr = fix(nSymbol/step);
        str=['Process on going>>',num2str(PerStr),'%'];
        waitbar(nSymbol/ Max_Symbols,hwait,str);
        pause(0.05);
    end

L = length(X);
A = zeros(V,K);
for k=1:V % Divided signals in frequency domain
    X into v non-overlapping sub-blocks
    A(k,:)=[zeros(1,(k-1)*L/V),X((k-1)*L/V+1:k*L/V),zeros(1,(V-k)*L/V)];
end
a = ifft(A,[],2);

% W=2
min_value = 10;
for n=1:Choose_Len1 % Applying optimum algorithm
    temp_phase = Phase_Set1(Choose1(n,:)).';
    temp_max(n) = max(abs(sum(a.*repmat(temp_phase,1,K))));% Represents the accumulation process
    if temp_max(n)<min_value
        min_value = temp_max(n);
        Best_n = n;
    End
end
aa = sum(a.*repmat(Phase_Set1(Choose1(Best_n,:)).',1,K));
Signal_Power = abs(aa.^2);
Peak_Power = max(Signal_Power,[],2);
Mean_Power = mean(Signal_Power,2);
PAPR_PTS2(nSymbol) = 10*log10(Peak_Power./Mean_Power);

% W=4
min_value = 10;
for n=1:Choose_Len2
    temp_phase = Phase_Set2(Choose2(n,:)).';
    temp_max(n) = max(abs(sum(a.*repmat(temp_phase,1,K))));
    if temp_max(n)<min_value
        min_value = temp_max(n);
        Best_n = n;
    End
end
aa = sum(a.*repmat(Phase_Set2(Choose2(Best_n,:)).',1,K));
Signal_Power = abs(aa.^2);
Peak_Power = max(Signal_Power,[],2);
Mean_Power = mean(Signal_Power,2);
PAPR_PTS4(nSymbol) = 10*log10(Peak_Power./Mean_Power);
end

[cdf1, PAPR1] = ecdf(PAPR_Original);
[cdf2, PAPR2] = ecdf(PAPR_PTS2);
[cdf3, PAPR3] = ecdf(PAPR_PTS4);

semilogy(PAPR1,1-cdf1,'linewidth',2)
hold on;
semilogy(PAPR2,1-cdf2,'c-*','linewidth',2)
hold on;
semilogy(PAPR3,1-cdf3,'r-d','linewidth',2)

legend(' Original', ' W=2', ' W=4')
xlabel('PAPR0 [dB]');
ylabel('CCDF (Pr[PAPR>PAPR0])');
axis([5 12 10e-4 1])
grid on
clc; clear all; close all;

N = 128; % The number of carriers
OF = 8;  % Oversampling factor
K = N*OF;

QPSK_Set = [1 -1 j -j];  % QPSK Constellation symbols
Phase_Set = [1 -1 j -j]; % Weighting factor
M = 4;  % The number of branches in SLM method
V = 4;  % The number of sub-blocks in PTS method

X1 = zeros(M,N); % Initialize the data matrix
Index1 = zeros(M,N);
X2 = zeros(1,N);
Index2 = zeros(1,N);
hwait = waitbar(0,'Please wait...'); % Creates and displays a waitbar

for i=1:4 % Generate all possible combinations of weighting factor set in PTS method
    X(i,1:4^i) = [repmat(1,1,4^(i-1)),repmat(2,1,4^(i-1)),repmat(3,1,4^(i-1)),repmat(4,1,4^(i-1))];
    Y = X(i,1:4^i);
    X(i,1:256) = repmat(Y,1,256/length(Y));
end

X = X.'; Choose = fliplr(X);
Choose_Len = 256; % The total number of combinations or IFFT operations in PTS method
Max_Symbols = 1e3; % The number of generated OFDM symbols

for nSymbol=1:Max_Symbols *10
    Index = randint(1,N,length(QPSK_Set)) + 1;
    X = QPSK_Set(Index(1,:)); % The QPSK modulation
    X = [X(1:N/2) zeros(1,K-N) X(N/2+1:N)]; % oversampling process
    x = ifft(X,[],2); % Signals in time domain after IFFT operation
    Signal_Power = abs(x.^2);
    Peak_Power = max(Signal_Power,[],2);
    Mean_Power = mean(Signal_Power,2);
    PAPR_Orignal(nSymbol) = 10*log10(Peak_Power./Mean_Power);
end;

step = Max_Symbols /100; % Set the parameters of waitbar
for nSymbol=1:Max_Symbols
    if Max_Symbols-nSymbol<=50
        waitbar(nSymbol/ Max_Symbols,hwait,'Almost done!');
    end;

else
    PerStr=fix(nSymbol/step);
    str=['Process on going>>>',num2str(PerStr),'%'];
    waitbar(nSymbol/Max_Symbols,hwait,str);
    pause(0.05);
end

%SLM
Index1(1,:) = randint(1,N,length(QPSK_Set))+1;
Index1(2:M,:) = randint(M-1,N,length(Phase_Set))+1;
X1(1,:) = QPSK_Set(Index1(1,:));
    % The QPSK modulation
Phase_Rot = Phase_Set(Index1(2:M,:));
X1(2:M,:) = repmat(X1(1,:),M-1,1).*Phase_Rot;
X11 = [X1(:,1:N/2) zeros(M,K-N) X1(:,N/2+1:N)];
    % oversampling process
x = ifft(X11,[1,2]);
    % Signals in time domain after IFFT operation
Signal_Power = abs(x.^2);
Peak_Power = max(Signal_Power,[1,2]);
Mean_Power = mean(Signal_Power,2);
PAPR_temp = 10*log10(Peak_Power./Mean_Power);
PAPR_SLM(nSymbol) = min(PAPR_temp);

%PTS
A = zeros(V,N);
    % Initial phase set to'
0',exp(j*0)
Index2 = randint(1,N,length(QPSK_Set))+1;
X2 = QPSK_Set(Index2(1,:));
Index= randperm(N);
for v=1:V
    % Divided signals in frequency domain X into V
    % non-overlapping sub-blocks
    A(v,Index(v:V:N)) = X2(Index(v:V:N));
end
A1 = [A(:,1:N/2) zeros(V,K-N) A(:,N/2+1:N)];
a = ifft(A1,[1,2]);
min_value = 10;
    % Applying optimum algorithm
for n=1:Choose_Len
    temp_phase = Phase_Set(Choose(n,:)).';
    temp_max = max(abs(sum(a.*repmat(temp_phase,1,K))));
    if temp_max<min_value
        min_value = temp_max;
        Best_n = n;
    End
end
aa = sum(a.*repmat(Phase_Set(Choose(Best_n,:)).',1,K));
    % Represent the accumulation process
Signal_Power = abs(aa.^2);
Peak_Power = max(Signal_Power,[1,2]);
Mean_Power = mean(Signal_Power,2);
PAPR_PTS(nSymbol) = 10*log10(Peak_Power./Mean_Power);
end
close(hwait);
[cdf1, PAPR1] = ecdf(PAPR_Original);
\[
\text{[cdf2, PAPR2]} = \text{ecdf(PAPR\_SLM)};
\]
\[
\text{[cdf3, PAPR3]} = \text{ecdf(PAPR\_PTS)};
\]

\text{semilogy(PAPR1,1-cdf1,'linewidth',2)}
\text{hold on;}
\text{semilogy(PAPR2,1-cdf2,'linewidth',2)}
\text{hold on;}
\text{semilogy(PAPR3,1-cdf3,'linewidth',2)}

\text{legend('Original',' SLM',' PTS');}
\text{xlabel('PAPR\_0 [dB]');}
\text{ylabel('CCDF (Pr[PAPR>PAPR\_0])');}
\text{axis([5 12 10e-4 1])}
\text{grid on}