A Mathematical Vernissage
- Illustrations by Karl H. Hofmann

Lunch Seminar
Karlskrona, 17/10 2017

Preview of the Talk

I. A Short Vita

II. Stars (rather long)

III. Marble Blocks

IV. The Quantum Labyrinth of the Minotaur

V. Schrödinger's Cat

VI. Minimal Surfaces

VII. Women in Academia

VIII. Rudi Völler

IX. Resonances

A Short Vita

Karl H. Hofmann was born on October 3, 1932 in Heilbronn, Germany.

He finished his Ph.D. in 1958 and his Habilitation in 1962 at the University of Tübingen.

In 1982 he moved to the Technical University, Darmstadt as a Professor until his retirement in 1998, but has remained active there as a Professor Emeritus, teaching courses and seminars.

He has made major contributions across the spectrum in topological algebra: continuous lattices and domain theory, rings and sheaves, C^*-algebras, Lie semigroups, the exponential function, transformation groups.

He is a dedicated teacher, commited to good pedagogy, and has been the advisor for thirty-seven Ph.D. students.

He is also a talented artist, and has done weekly illustrated poster announcements of the Darmstadt colloquia, a front-page for the AMS Notices and illustrated the well-known Proofs from the Book.

Fika is very important to him and he loves good cake!


PROF. DR. SIEGFRIED ECHTERHOFF
UNIVERSITÄT MÜNSTER 3. DEZEMBER 03
NICHTKOMMUTATIVE ALGEBRAISCHE
TOPOLOGIE UND DIE BAUM-CONNES
VERMUTUNG

The Star of Bethlehem

The Star of Bethlehem, or Christmas Star, appears in the story of the Gospel of Matthew, where "wise men from the East" are inspired by the star to travel to Jerusalem, which leads them to Jesus'.

Many Christians believe that the star was a miraculous sign to mark the birth of the Christ (or Messiah). Some theologians claim that the star fulfilled a prophecy, known as the Star Prophecy.

The Star of an Algebra

In the early 20th century ”wise men from the East” (u.a. von Neumann, Kaplansky, Gelfand, and Naimark) realized the full power of so-called *-algebras, which lead them, e.g., to the mathematical foundation of quantum mechanics and the rich theory of so-called C*-algebras.

Definition: *-Algebras

A *-algebra, or involutive algebra, is a pair $(\mathcal{A}, ^*)$ consisting of a complex algebra $\mathcal{A}$ and an antilinear map $^* : \mathcal{A} \to \mathcal{A}$ of order 2, i.e.,

(i) $(a+b)^* = a^* + b^*$ for all $a, b \in \mathcal{A}$.

(ii) $(z \cdot a)^* = \overline{z} \cdot a^*$ for all $z \in \mathbb{C}$ and $a \in \mathcal{A}$.

(iii) $(a^*)^* = a$ for all $a \in \mathcal{A}$.

Examples:

1. The complex numbers $\mathbb{C}$ together with its canonical multiplication and conjugation

$$z^* := \overline{x + \imath y} = x - \imath y = \overline{z}$$

define a *-algebra.

2. Let X be a topological space. Then the space $C(X)$ of continuous functions on X with pointwise multiplication and conjugation

$$f^*(x) := \overline{f(x)} \quad \text{for all} \quad x \in X.$$

3. For every $n \in \mathbb{N}$ the space $\mathrm{M}_n(\mathbb{C})$ of $\mathbb{C}$-valued $n \times n$-matrices together with its canonical multiplication and conjugation

$$M^* := \overline{M}^T$$

define a *-algebra.

4. Let $\mathcal{H}$ be a Hilbert space. Then the space $B(\mathcal{H})$ of bounded linear operators on $\mathcal{H}$ together with composition of operators and conjugation by taking adjoints, i.e., given $T \in B(\mathcal{H})$ define T^* by

$$\langle Tx, y \rangle = \langle x, T^* y \rangle \quad \text{for all} \quad x, y \in \mathcal{H},$$

is a *-algebra.

5. Let G be a group. The the corresponding group algebra $\mathbb{C}[G]$, i.e., the free vector space on G, together with its convolution product and conjugation

$$a^* := \left(\sum_{g \in G} a_g \cdot g \right)^* := \sum_{g \in G} \overline{a_g} \cdot g^{-1}$$

is a *-algebra.

More Definitions: Let $(\mathcal{A},^*)$ be a *-algebra.

(i) An element $e \in \mathcal{A}$ is called an *idempotent* if $e^2 = e$.

(ii) An idempotent $p \in \mathcal{A}$ is called *self-adjoint* if $p = p^*$.

Examples:

1. The only idempotents in $\mathbb{C}$ are 0 and 1.

2. A topological space X is connected if and only if 0 and 1 are the only idempotents in $C(X)$.

3. There are plenty of idempotents in $B(\mathcal{H})$. In fact, there is a 1:1 correspondence between self-adjoint idempotents in $B(\mathcal{H})$ and closed subspaces of $\mathcal{H}$.

Some Interesting Prophecies:

1. Each idempotent $e \in \mathbb{C}[G]$ is essentially self-adjoint, that is, there is a self-adjoint idempotent $p \in \mathbb{C}[G]$ with $e\mathbb{C}[G] = p\mathbb{C}[G]$. Geometrically, this means that the space $e\mathbb{C}[G]$ has an orthogonal complement.

2. (Gospel of Kaplansky) If G is a torsion-free group, then every idempotent in $\mathbb{C}[G]$ is trivial, i.e., equal to 0 or 1.

3. (Gospel of Baum–Connes) The Baum-Connes prophecy suggests a link between the K-theory of the reduced C*-algebra of a group and the K-homology of the classifying space of proper actions of that group, i.e., an interesting correspondence between different areas of mathematics such as geometry and analysis.

Note: Two humble, modest, and cake loving disciples at BTH work on 1. and 2.

Back to Karl. H. Hofmann:

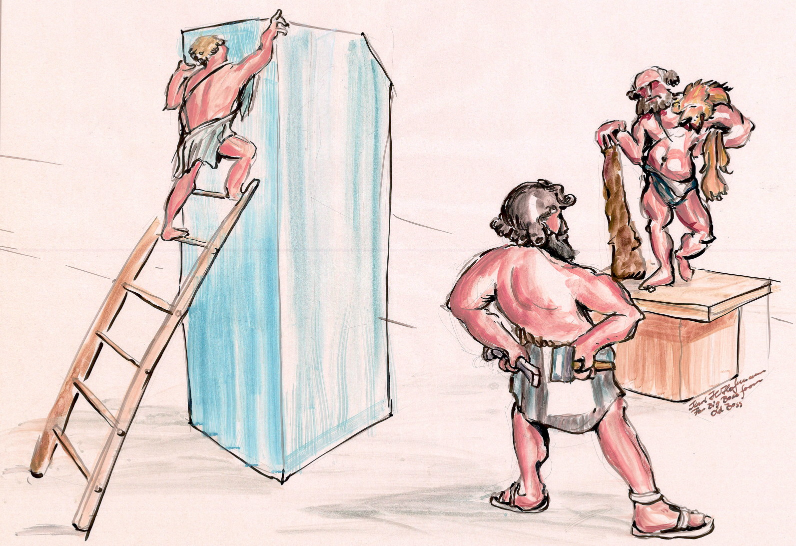
By a famous Theorem of Gelfand and Naimark each commutative C*-algebra is isomorphic to $C(X)$ for some compact space X.

$\rightsquigarrow$ Consider noncommutative C*-algebras as functions on a "quantum space".

However, one might hope for a more rigorous characterization:

The Dauns–Hofmann Theorem (mid-60s):

Each C*-algebra is a module over the algebra of continuous functions on its primitive ideal space.



Prof. Dr. Karl-Hermann Neeb

KONVEXITÄT VND EXTRE-MALPVNKTE IN DER DARSTELLVNGSTHEORIE

Antrittsvorlesung an der Technischen Universität Darmstadt am 21. 10. 1998

Praxiteles the Sculptor, 4th century BC

The poster is dedicated to Karl H. Hofmanns successor K.-H. Neeb (my Ph.D. supervisor).

On the poster you can see Praxiteles, who has to cut out the statue of Hercules from a gigantic convex marble block.

Next to it is a somewhat skinny gentleman, who acts as a model.

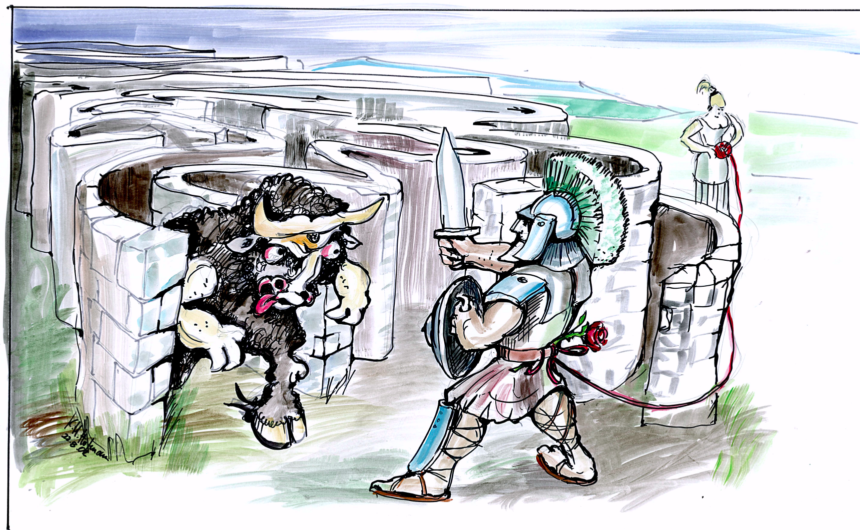
The illustrator might want to indicate that it is not yet clear what kind of (mathematical) Hercules the block will become.

Extreme Points and Convexity in Representation Theory

Given a unital C*-algebra $\mathcal{A}$, its states, i.e., positive linear functionals of norm 1, form a compact convex set whose extreme points correspond to irreducible representations.

If, for example, $\mathcal{A}$ is the algebra $C(X)$ of continuous functions on a compact space, then its states are probability measures on X and its extreme points are Dirac measures.

The philosophy underlying these facts can be adapted to the representation theory of infinite-dimensional Lie groups (such as the unitary group of a C*-algebra).



Prof. Dr. Burkhard Kümmerer
Technische Univ. Darmstadt
Antrittsvorlesung
Auf Irrwegen sicher zum Ziel
Aspekte der Quantenwahr-
scheinlichkeitstheorie

23. Oktober 2002

The Quantum Labyrinth of the Minotaur

The Minotaur, being part man and part bull, dwelts at the center of an elaborated Labyrinth.

Ariadne, the daughter of King Minos, gives her beloved Athenian hero Theseus a ball of thread, so he can kill the Minotaur and find his way out of the Labyrinth.

The wavy walls most likely represent the wave function describing the quantum state of the labyrinth.

Quantum Probability

Quantum probability was developed in the 1980s as a noncommutative analog of the Kolmogorovian theory of stochastic processes.

In classical probability theory, information is summarized by the σ-algebra Σ of events in a classical probability space (X, Σ, μ).

Quantum information is described in similar algebraic. In fact, the appropriate algebraic structure for observables is a *-algebra.

Definition: Quantum Probability Space

A *quantum probability space* is a pair $(\mathcal{A}, \phi)$ consisting of a *-algebra $\mathcal{A}$ and a state ϕ.

Remark:

Every classical probability space gives rise to a quantum probability space if $\mathcal{A}$ is chosen as the *-algebra of bounded measurable $\mathbb{C}$-valued functions on it.

The projections $p \in \mathcal{A}$ are the events in $\mathcal{A}$, and $\phi(p)$ gives the probability of the event p.

Prof. Dr. Jürg Fröhlich, Eidgenössische Technische Hochschule Zürich

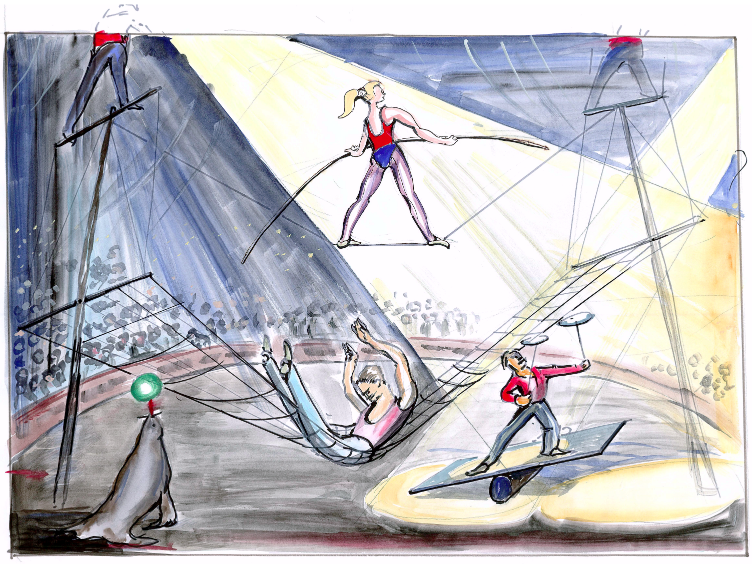
Der Zufall in der Quantenmechanik. 20. Mai 2015. Tee 16:45

Schrödinger's Cat

Schrödinger's cat is a famous thought experiment devised by Austrian physicist Erwin Schrödinger in 1935.

The scenario presents a cat that may be simultaneously both alive and dead, a state known as a quantum superposition.

It poses the question, "when does a quantum system stop existing as a superposition of states and become one or the other?"



ANTRITTSVORLESUNG

von PROF. DR. KARSTEN GROSSE-BRAUCKMANN

TU DARMSTADT

GEOMETRIE VON GLEICHGEWICHTSFLÄCHEN

13. FEBRUAR 2002

Minimal Surfaces

Minimal surfaces can be defined in several equivalent ways in $\mathbb{R}^3$ and lie at the crossroads of different mathematical disciplines, especially differential geometry, calculus of variations, potential theory, complex analysis and mathematical physics.

Definition: Differential Equation

A surface $M \subseteq \mathbb{R}^3$ is minimal if and only if it can be locally expressed as the graph of a solution of

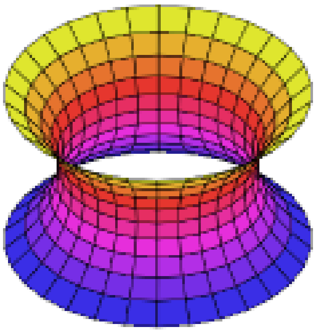
$$(1 + u_x^2)u_{yy} - 2u_x u_y u_{xy}(1 + u_y^2)u_{xx} = 0.$$

Definition: Mean Curvature

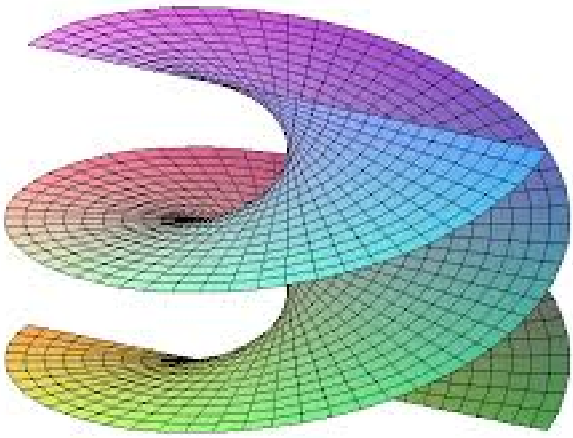
A surface $M \subseteq \mathbb{R}^3$ is minimal if and only if its mean curvature vanishes identically.

Examples:

1. The plane, of course.



2. The Catenoid.



3. The Helicoid.

Remark:

Minimal surfaces are part of the generative design toolbox used by modern designers.

In architecture there has been much interest in tensile structures, which are closely related to minimal surfaces.

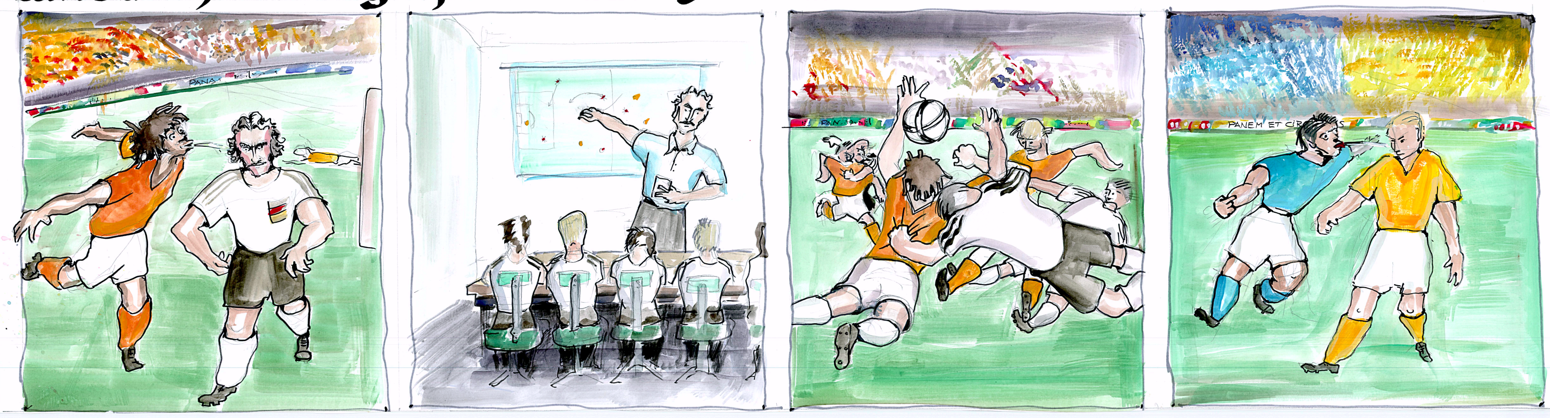
A famous example is the Olympiapark in Munich by Frei Otto, inspired by soap surfaces.


Professor Dr.
Andrea Blunck
Doktortitel
und nicht weiter
Frauen in der
Mathematik
Gestern
Heute
Morgen
27.11.08
18:00
Im Schloss
S3 13/36
TUD
100 Jahre Frauenstudium

Ph.D. and not Further.
Women in Academia.

Only 17 percent of professors in Germany - 6.700 out of 38600 in total - were female in 2010. In mathematics it is even worse.

Prof. Dr. Timo Leuders, Pädagogische Hochschule Freiburg: Kooperatives Problem
lösen mit realistischen Aufgaben - deutsche Erfahrungen mit der nieder-
ländischen A-lympiade 30. Juni 2004



Cooperative problem-solving with realistic tasks - German experiences with the Dutch A-lympiade

The A-lympiad is a mathematical contest for teams of 3 or 4 students organized by the Freudenthal institute of Utrecht University in the Netherlands.

The teams work on an assignment in which mathematical problem solving and higher order thinking skills must be used to solve a real world problem.

Prof. Dr. Joachim Hilgert
Universität Paderborn

Resonanzen

13. Januar 2015. In 244
Tee um 16:45

Thank you for your attention!

I'd be happy about any kind of resonance.