Fuzzy One-Decision Making Model with Fuzzified Outcomes in the Treatment of Necrotizing Fasciitis

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The agenda of the presentation

- The model of fuzzy decision making with the utility matrix
- The outline of fuzzy one-decision making
- The construction of entry data
- The structure of fuzzified outcomes
- The recommendations for treating the Necrotizing Fasciitis (NF) with Hyperbaric Oxygen (HBO)
- Conclusions

The aim of the current research

- Necrotizing fasciitis (NF) is a rare, but deadly soft tissue infection, usually treated with antibiotics and surgery. The treatment with hyperbaric oxygen (HBO) can be helpful.
- We want to know, if the patient has a good prognosis of recovery without the treatment with HBO or he/she needs the HBO supplement.
- To make the prognosis, a physician relies on his experience. The number of patients is little, which makes difficult to solve routinely the problems of HBO dosing.
- We thus initialize the mathematical model of fuzzy decision making, which considers only one decision (indications of treating the patient with HBO).
- The decision is differentiated in recommendation levels.
- We arrange recommendations in two families of terms: “stages of non-indication” versus “stages of indication”. These are converted to two families of fuzzy sets with parametric membership functions.
- We also introduce fuzzy sets, assigned to symptoms. By cumulating the symptoms intensities, we find the patient’s clinical characteristics. To accept the most convincing recommendation for HBO dosing, the cumulated characteristics of the patient will be tested in all decision levels.

The conception of a continuous fuzzy set

Suppose that symptom X_j is a fuzzy set.

A fuzzy set $X_j \subset X$ consists of two parts: the space of elements belonging to the set and the information about how strongly elements are related to the set, when referring to its definition.

To evaluate the relationship between elements and the set we design the set's membership function. The membership function μ_{Xj} of a fuzzy set X_j is a function defined as

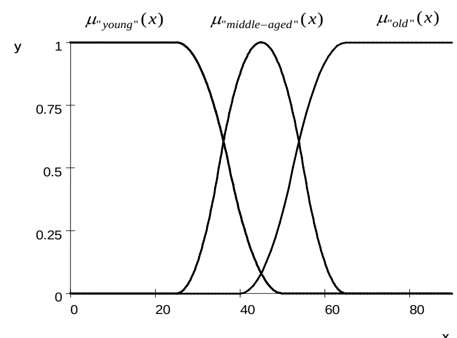
$$\mu_{Xj} : X \to [0,1].$$

Every element $x \in X$ has *a membership degree* $\mu_{Xj}(x) \in [0,1]$. The fuzzy set X_j is completely determined by the set of pairs

$$X_j = \{(x, \mu_{X_j}(x)) : x \in X\}$$

or

$$X_j = \sum_{x \in X} \mu_{X_j}(x) / x$$



The support of a fuzzy set X_j, supp(X_j), is a set of all $x \in X$, such that $\mu_{X_j}(x) > 0$.

Classical fuzzy decision making with a utility matrix

- A space of states (e.g., symptoms) $X=\{x_1,\ldots,x_n\}$, a decision space (e.g., treatments) $D=\{d_1,\ldots,d_d\}$ and a utility matrix U, are the items of fuzzy decision making.

$$U=\begin{array}{c} \\ d_1 \\ \vdots \\ d_d \end{array}\begin{array}{c} \begin{array}{ccc} x_1 & \cdots & x_n \end{array} \\ \begin{bmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & \ddots & \vdots \\ u_{d1} & \ldots & u_{dn} \end{bmatrix} \end{array}$$

has the entries u_{bj}, b = 1,…,d, j = 1,…,n. Each u_{bj} is the fuzzy utility of applying decision d_b to state x_j. In most of applications, u_{bj} are evaluated intuitively as values from interval [0, 1], e.g., utility of (d_1,x_1) = 0.7. Some users prefer determining the utilities as fuzzy sets, e.g., utility of (d_1,x_1) = "large".

- The aggregated utility U_{d_b} of d_b was estimated as $U_{d_b}=\sum_{j=1}^{n}u_{bj}$ in the early trials of adapting fuzzy decision making to practical solutions.
- The operation $\max(U_{d_1},\ldots,U_{d_d})$ allowed selecting the optimal d_b, satisfying the maximum criterion.

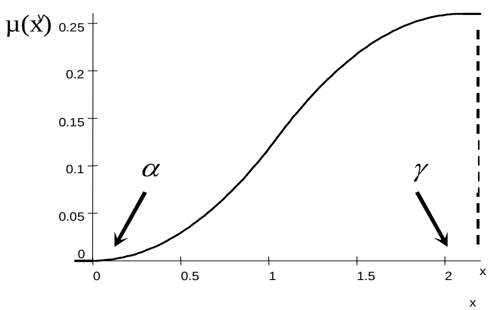

The general description of fuzzy one-decision model

- We still keep the set of symptoms $X = \{x_1,...,x_n\}$.
- For patient P_i, $i = 1,\ldots,p$, we sample the characteristics s_i, informing about presence or absence of symptoms X_j, $j = 1,\ldots,n$, typical of necrotizing fasciitis.
- Each clinical marker X_j, $j = 1,\ldots,n$, is replaced by a fuzzy set X_j. A marker value $x_{i,j}$ for symptom X_j will have the membership degree $\mu_{X_j}(x_{i,j})$.
- The importance weights w_j of symptoms X_j are suggested by the physician as the sequence $X_1 \succ \ldots \succ X_n$, where "$\succ$" means "$X_j$ emerges more dangerous impact on the patient health state than X_h, j, $h = 1,\ldots,n$. We state $w_1 > \ldots > w_n$ and want $\sum_{j=1}^{n} w_j = 1$.
- Patient characteristics $s_i = \sum_{j=1}^{n} \mu_{X_j}(x_{i,j}) \cdot w_j,\; i = 1, \ldots, p$, belongs to [0, 1].
- Sets L_l, $l = 1,\ldots,m$, are fuzzy sets, assisting recommendation levels.
- We test s_i in each L_l by computing $\mu_{L_l}(s_i)$. The optimal decision level L^* satisfies $\mu_{L^*}(s_i) = \max_{1 \le l \le m}(\mu_{L_l}(s_i))$.

$$\begin{bmatrix} \mu_{X_1}(x_{i,1}) \cdot w_1 \\ \vdots \\ \mu_{X_n}(x_{i,n}) \cdot w_n \end{bmatrix} \to s_i = \sum_{j=1}^{n} \mu_{X_j}(x_{i,j}) \cdot w_j \to \begin{pmatrix} \mu_{L_1}(s_i) \\ \vdots \\ \mu_{L_m}(s_i) \end{pmatrix} \to d = L^* \text{for which}\, \mu_{L^*}(s_i) = \max_{1 \le l \le m} \mu_{L_l}(s_i).$$

Fuzzy sets of measurable symptoms

- Symptoms X_j are recognized as quantitative and qualitative features. We assign fuzzy sets X_j, $j = 1,\ldots,n$, to both types.
- As the rising order of symptom values (real values or codes) is associated with the growing states of the disease threat then the membership functions of X_j will be constructed as ascending functions.
- Measurable symptoms X_j, taking values $x_{i,j}$ in interval [α, γ] continuously, have the membership function $\mu_{X_j}(x_{i,j})$ as a parametric *s*-function $s(x_{i,j}, \alpha, \beta, \gamma)$, yielded by



$$\mu_{X_j}(x_{i,j}) = s(x_{i,j},\alpha,\beta,\gamma) = \begin{cases} 0 & \text{for } x_{i,j} \le \alpha, \\ 2\left(\frac{x_{i,j}-\alpha}{\gamma-\alpha}\right)^2 & \text{for } \alpha < x_{i,j} \le \beta, \\ 1-2\left(\frac{x_{i,j}-\gamma}{\gamma-\alpha}\right)^2 & \text{for } \beta < x_{i,j} \le \gamma, \\ 1 & \text{for } x_{i,j} > \gamma, \end{cases}$$

where $\beta = (\alpha+\gamma)/2$, $j = 1,\ldots,n$, $i = 1,\ldots,p$, $x_{i,j} \in \text{supp}(X_j)$.

Example 1

Symptom "*age*"= X_2 is a fuzzy set, constrained by the membership function $s(x_{i,2}, 18, 59, 100)$. For, e.g., $x_{i,2} = 76$, we estimate $\mu_{X_2}(76) = 1 - 2\left(\frac{76-100}{100-18}\right)^2 = 0.828$ in accordance with the condition 59<76<100.

Fuzzy sets of compound qualitative symptoms

- Compound qualitative symptoms X_j, have a list of codes $C_{X_j} = \{0,\ldots,k,\ldots,z\}$, where $k = 0,\ldots,z$, are non-negative integers. Suppose that z is an even integer.
- k-values mark answers to a question about the intensity of symptom X_j in P_i. We suppose that 0 denies the presence of X_j, z confirms X_j's critical stage, and $(0+z)/2$ indicates "*medium intensity*".
- Function $g(k)$, starting with $g(0) = -1$ and terminating with $g(z) = 1$ has an equation

$$g(k) = g(0) + k \cdot \tfrac{g(z)-g(0)}{z} = -1 + k \cdot \tfrac{2}{z}$$

- Interval [–1, 1], with discrete values $g(k)$, is a support of fuzzy set X_j.
- The membership function of X_j is given by

$$\mu_{X_j}(g(k)) = s(g(k),-1,0,1) = \begin{cases} 2\left(\frac{g(k)+1}{2}\right)^2 & \text{for } -1 \le g(k) \le 0, \\ 1-2\left(\frac{g(k)-1}{2}\right)^2 & \text{for } 0 \le g(k) \le 1. \end{cases}$$

Example 2

Levels of symptom "*medical state*" = S_1 are coded as: "*comfortable*" = 0, "*satisfactory*" = 1, "*stable*" = 2, "*critical but stable*" = 3, and "*critical*" = 4. For $k = 0,\ldots,4$, $g(0) = -1 + 0 \cdot \tfrac{2}{4} = -1, g(1) = -0.5$ $g(2) = 0$, $g(3) = 0.5$, and $g(4) = 1$. The membership degrees, found for $g(k)$, $k = 0,\ldots,4$, are $\mu_{X_1}(g(0)) = 0$, $\mu_{X_1}(g(1)) = 0.125$, $\mu_{X_1}(g(2)) = 0.5$, $\mu_{X_1}(g(3)) = 0.875$, and $\mu_{X_1}(g(4)) = 1$.


The importance weights w_j of symptoms X_j

- By "importance" we mean the strength of X_j's harmful power in the running process of the illness.
- Generally, we arrange n symptoms X_j in the sequence $X_1 \succ \ldots \succ X_n$ in accordance with the expert's opinion. We want the sum of all weights w_j, $j = 1,\ldots,n$, to be 1.

$$n \cdot r + (n-1) \cdot r + \ldots + 2 \cdot r + 1 \cdot r = 1$$

where r is a quotient dependent on n.

$$w_j = (n - j + 1) \cdot r, \text{ for } j = 1,\ldots,n.$$

Example 3

The symptoms in necrotizing fasciitis are listed in the importance order, as
"*medical state*" = $X_1 \succ$ "*age*" = $X_2 \succ$ "*risk factors*" = $X_3 \succ$ "*crp*" = $X_4 \succ$ *wbc* = $X_5 \succ$ "*temperature*" = X_6.
"*crp*" stands for C-reactive proteins and "*wbc*" means white blood cells.
Equation $6r+5r+4r+3r+2r+r = 1$ provides $r = 0{,}0476$.
We receive, in turn for $j = 1,\ldots,6$, the weights
$w_1 = (6–1+1)0.0476 = 0.2856$, $w_2 = 0.238$, $w_3 = 0.1904$, $w_4 = 0.1428$, $w_5 = 0.0952$, $w_6 = 0.0476$.


The assumptions for constructions of fuzzified output levels

- We generate output decision fuzzy levels L_l over [0, 1], $l = 1,\ldots,m$, to calculate the membership degrees of patient characteristics s_i in each L_l for patient P_i.
- L^*, which is the optimal decision level assigned to P_i, satisfies

$$\mu_{L^*}(s_i) = \max_{1 \le l \le m} \mu_{L_l}(s_i).$$

- We suppose that m is an even positive arbitrary integer in the model, due to the assumption that recommendation levels are differentiated into families: “non-indications” and “indications.
- Membership functions of L_l are dependent only on a number of levels and a length of the common set containing all supports of L_l.


Membership functions for an even number of output levels

For an even *m* value, we divide all sets in two families.

“*left*” sets $L_1,\ldots,L_{\frac{m}{2}}$

$$\mu_{L_t}(s_i)=\begin{cases}1 & \text{for } s_i \le \frac{E\cdot(m-2)}{2(m-1)}\delta(t),\\ 1-2\left(\dfrac{s_i-\frac{E\cdot(m-2)}{2(m-1)}\delta(t)}{\frac{E}{(m-1)}\delta(t)}\right)^2 & \text{for } \frac{E\cdot(m-2)}{2(m-1)}\delta(t)\le s_i \le \frac{E}{2}\delta(t),\\ 2\left(\dfrac{s_i-\frac{E\cdot m}{2(m-1)}\delta(t)}{\frac{E}{(m-1)}\delta(t)}\right)^2 & \text{for } \frac{E}{2}\delta(t)\le s_i \le \frac{E\cdot m}{2(m-1)}\delta(t),\\ 0 & \text{for } s_i \ge \frac{E\cdot m}{2(m-1)}\delta(t),\end{cases}$$

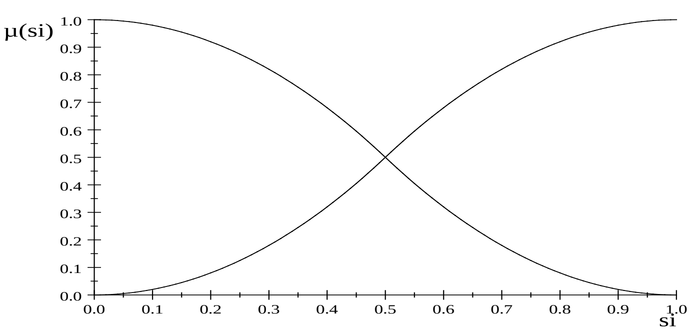
$$\delta(t)=\frac{2}{m}\cdot t\,,\ t=1,\ldots,\frac{m}{2}\,.$$

“*right*” sets $L_{\frac{m}{2}+1},\ldots,L_m$

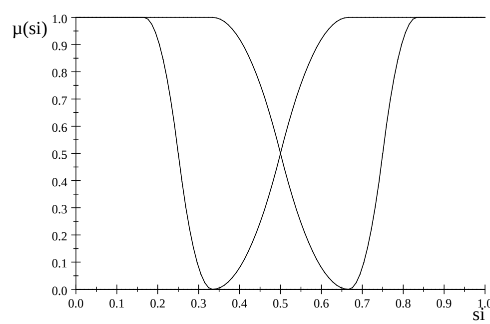
$$\mu_{L_{\frac{m}{2}+t}}(s_i)=\begin{cases}0 & \text{for } s_i \le E-\frac{E\cdot m}{2(m-1)}\varepsilon(t),\\ 2\left(\dfrac{s_i-\left(E-\frac{E\cdot m}{2(m-1)}\varepsilon(t)\right)}{\frac{E}{(m-1)}\varepsilon(t)}\right)^2 & \text{for } E-\frac{E\cdot m}{2(m-1)}\varepsilon(t)\le s_i \le E-\frac{E}{2}\varepsilon(t),\\ 1-2\left(\dfrac{s_i-\left(E-\frac{E\cdot(m-2)}{2(m-1)}\varepsilon(t)\right)}{\frac{E}{m-1}\varepsilon(t)}\right)^2 & \text{for } E-\frac{E}{2}\varepsilon(t)\le s_i \le E-\frac{E\cdot(m-2)}{2(m-1)}\varepsilon(t),\\ 1 & \text{for } s_i \ge E-\frac{E\cdot(m-2)}{2(m-1)}\varepsilon(t),\end{cases}$$

$$\varepsilon(t)=1-\frac{2}{m}(t-1)\,,\ t=1,\ldots,\frac{m}{2}\,.$$

m=2,E=1, $\delta(1)=1,\ \varepsilon(1)=1,$



m=4,E=1, $\delta(1)=0.5,\ \delta(2)=1,\ \varepsilon(1)=1,\ \varepsilon(2)=0.5$



Fuzzy sets of recommendation levels

Example 4

The term list of decision

d = "recommendation for treating with HBO for patient P_i" is stated as

d = {L_1 = strong non-indication for treating with HBO", L_2 = moderate non-indication for treating with HBO", L_3 ="moderate indication for treating with HBO", L_4 = "strong indication for treating with HBO".

L_1 and L_2 belong to the "*left*" family of fuzzy sets, whereas L_3 and L_4 build the "*right*" family of fuzzy sets. Sets L_l have the supports included in interval [0, 1], due to the statement $s_i \in [0, 1]$. For $m = 4$ and $E = 1$, we get

$$\mu_{L_1}(s_i) = \begin{cases} 1 & \text{for } 0 \le s_i \le 0.166, \\ 1-2\left(\frac{s_i-0.166)}{0.166}\right)^2 & \text{for } 0.166 \le s_i \le 0.25, \\ 2\left(\frac{s_i-0.333}{0.166}\right)^2 & \text{for } 0.25 \le s_i \le 0.333, \\ 0 & \text{for } s_i \ge 0.333, \end{cases}$$

$$\mu_{L_2}(s_i) = \begin{cases} 1 & \text{for } 0 \le s_i \le 0.333, \\ 1-2\left(\frac{s_i-0.333)}{0.333}\right)^2 & \text{for } 0.333 \le s_i \le 0.5, \\ 2\left(\frac{s_i-0.666}{0.333}\right)^2 & \text{for } 0.5 \le s_i \le 0.666, \\ 0 & \text{for } s_i \ge 0.666, \end{cases}$$

$$\mu_{L_3}(s_i) = \begin{cases} 0 & \text{for } 0 \le s_i \le 0.333, \\ 2\left(\frac{s_i-0.333}{0.333}\right)^2 & \text{for } 0.333 \le s_i \le 0.5, \\ 1-2\left(\frac{s_i-0.666}{0.333}\right)^2 & \text{for } 0.5 \le s_i \le 0.666, \\ 1 & \text{for } s_i \ge 0.666, \end{cases}$$

$$\mu_{L_4}(s_i) = \begin{cases} 0 & \text{for } 0 \le s_i \le 0.667, \\ 2\left(\frac{s_i-0.667}{0.167}\right)^2 & \text{for } 0.667 \le s_i \le 0.75, \\ 1-2\left(\frac{s_i-0.833}{0.167}\right)^2 & \text{for } 0.75 \le s_i \le 0.833, \\ 1 & \text{for } s_i \ge 0.833. \end{cases}$$

Treatment with HBO – membership of values of clinical markers

- The values of crucial clinical markers and the decisions made by physicians have been sampled for 13 patients (12 men and 1 woman) treated in the Blekinge County City Hospital in Karlskrona, Sweden, between 2006 and 2010.
- The clinical symptoms, essential in NF, have been introduced in Example 3. For quantitative symptoms we adapt $\mu_{X_j}(x_{i,j}) = s(x_{i,j}, \alpha_j, \beta_j, \gamma_j)$ as follows:

  $\mu_{X_2="age"}(x_{i,2}) = s(x_{i,2}, 18, 59, 100)$, $\mu_{X_4="crp"}(x_{i,4}) = s(x_{i,4}, 0, 250, 500)$,
  $\mu_{X_5="wbc"}(x_{i,5}) = s(x_{i,5}, 0, 15, 30)$, and $\mu_{X_6="temp."}(x_{i,6}) = s(x_{i,6}, 36, 38.5, 41)$.
- In Example 2, we have determined the membership degrees for the coded symptom X_1 = "*medical state*" as: $\mu_{X_1}(g(0)) = 0$, $\mu_{X_1}(g(1)) = 0.125$, $\mu_{X_1}(g(2)) = 0.5$, $\mu_{X_1}(g(3)) = 0.875$, and $\mu_{X_1}(g(4)) = 1$.

  Another symptom X_3 = "*risk factors*", coded between 0 and 6, is characterized by memberships

  $\mu_{X_3}(g(0)) = 0$, $\mu_{X_3}(g(1)) = 0.056$, $\mu_{X_3}(g(2)) = 0.221$, $\mu_{X_3}(g(3)) = 0.5$,
  $\mu_{X_3}(g(4)) = 0.779$, $\mu_{X_3}(g(5)) = 0.944$, and $\mu_{X_3}(g(6)) = 1$.

Patient symptom values and membership degrees in fuzzy sets designed for symptoms X_j, $j = 1,\ldots,6$

P_i	X_1-medical state	X_2-age	X_3-risk factors	X_4-crp	X_5-wbc	X_6-temperature
P_1	0.13/1	0.06/32	0/0	0.83/352	0.52/15.3	0/36.2
P_2	0/36.2	0.83/76	0.5/3	0.56/267	0.49/14.9	0.39/38.2
P_3	0.88/3	0.30/50	0.06/1	0.43/232	0.22/10	0.14/37.3
P_4	0.5/2	0.66/66	0.22/2	0.7/305	0.99/28.2	0.29/37.9
P_5	0.88/3	0.75/71	0/0	0.29/189	0.99/27.8	0.14/37.3
P_6	0.5/2	0.45/57	0/0	0.64/281	0.53/15.5	0.42/38.3
P_7	1/4	0.29/49	0.06/1	0.85/363	0.76/19.5	0.03/36.6
P_8	0.88/3	0.89/81	0.5/3	0.91/394	0.36/12.7	0.20/37.6
P_9	1/4	0.48/58	1/6	0.94/413	0.68/18	0.32/38
P_{10}	0.88/3	0.45/57	0.06/1	0.48/246	0.02/3.1	0/35.8
P_{11}	0.5/2	0.52/60	0.22/2	0.06/85	0.62/16.9	0.29/36.5
P_{12}	0.88/3	0.73/70	0.78/4	0.92/403	0.99/28.5	0.32/38
P_{13}	1/4	0.88/80	0.22/2	0.05/76	0.73/18.9	0.98/40.5


Assumptions for making decisions about serving HBO

Example 5

Patient P_1 is represented by

$s_1 = 0.125\cdot0.286+0.058\cdot0.238+0\cdot0.19+0.824\cdot0.1428+0.52\cdot0.095+0.003\cdot0.047 = 0.217$.

For s_i = 0.217, we get: $\mu_{L_1}(0.217) = 1 - 2\left(\frac{(0.217-0.166)}{0.166}\right)^2 = 0.28$ (0.166 <0.217 <0.25), $\mu_{L_2}(0.217) = 1$ (0.217 < 0.333), $\mu_{L_3}(0.217) = 0$ (0.217 < 0.333), and $\mu_{L_4}(0.217) = 0$ (0.217 < 0.667).

The largest value of the membership degree indicates level L_2.

Recommendations for HBO treating - the Comparison of Fuzzy Decisions (Underlined) to Decisions Made by the Physician

P_i	s_i	$\mu_{L_1}(s_i)$	$\mu_{L_2}(s_i)$	$\mu_{L_3}(s_i)$	$\mu_{L_4}(s_i)$	Decision made by physician
P_1	0.217	0.81	1	0	0	No
P_2	0.58	0	0.13	0.87	0	Yes
P_3	0.42	0	0.86	0.14	0	No
P_4	0.558	0	0.25	0.75	0	Yes
P_5	0.57	0	0.17	0.83	0	Yes
P_6	0.41	0	0.89	0.11	0	No
P_7	0.56	0	0.21	0.79	0	Yes
P_8	0.73	0	0	1	0.29	Yes
P_9	0.80	0	0	1	0.93	Yes
P_{10}	0.44	0	0.8	0.2	0	No
P_{11}	0.39	0	0.94	0.06	0	No
P_{12}	0.81	0	0	1	0.97	Yes
P_{13}	0.66	0	0	1	0	Yes

Conclusion and Future Work

- We have used our model to advise the treatment with hyperbaric oxygen in necrotizing fasciitis.
- Instead of designing a utility matrix, we have introduced only one decision, designated by the list of term-sets. These express recommendation levels of the treatment as non-indications and indications.
- The decision levels are involved in the algorithm in its final phase, which differs the model from other fuzzy decision making models.
- Our decisions are made for individuals, and they have not general characters.
- The input data and output recommendation levels are fuzzified by designs of own suggestions of membership functions.
- The membership functions of the outcomes (recommendation levels) are sampled in two common formulas, depending only on a number of recommendation terms and the width of a reference set, common for terms.
- The own procedures of estimating the importance weights of symptoms and approximating membership degrees of qualitative symptoms have also been added as contributions in imprecise mathematics.
- The idea of combining analysis of numerical parameters with the qualitative estimations is very promising for the real decision making progress.
- The decisions have "*softer*" character than two-valued decisions "*yes-no*".
- We will redefine the ordering of importance weights of symptoms to refine the results. We also want to add the middle level "*wait and see*".
- The emphasis is laid on the design of recommendation levels, which classifies the model as robust approach to algorithmic modeling of outcomes.

Thank you for your attention!